95.141 Exam 1, October 5, 2009

Section Number ______________

Section Instructor_____________

Name ___________________________,____________________
Last Name First Name

Last 3 Digits of Student ID number: ____ ____ ____

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers! Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. **You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.**

Score on each problem:

1. _______

2. _______

3. _______

4. _______

Total Score (based on 100 pts) ________

Be prepared to show your student ID Card

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Problem 1 (25 points – 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

1-1. You are given an (incorrect) expression for position as a function of time:

$$x(t) = \frac{Bvt}{ma}$$

If a=acceleration in *m/s*^{2}, m=mass in *kg*, v=speed in *m/s*, and t=time in *s*, what do the units of the constant B have to be to make the equation dimensionally correct (have the correct units)?

A) $[m]$ B) $\left[\frac{mkg}{s}\right]$ C) $\left[\frac{mkg}{s^2}\right]$ D) $\left[\frac{kg}{s^2}\right]$

1-2. $\vec{V}_1 = 4\hat{i} + 7\hat{j} - 2\hat{k}$, $\vec{V}_2 = -6\hat{i} + 5\hat{j} + 4\hat{k}$, and $\vec{V}_1 - \vec{V}_2 + \vec{V}_3 = \hat{i} + 2\hat{j} - 3\hat{k}$. What is $\vec{V}_3$?

A) $-9\hat{i} + 3\hat{k}$ B) $3\hat{i} - 10\hat{j} - 5\hat{k}$ C) $-\hat{i} + 14\hat{j} - \hat{k}$ D) $11\hat{i} + 4\hat{j} - 9\hat{k}$

1-3. A hiker walks 2 miles to the East, then 4 miles North, then 2 miles West. The hiker walks with a constant speed, and the total walk takes her 2.5 hrs. What is the hiker's average velocity in miles per hour (mph)?

A) 2 mph North B) 4 mph North C) 1.6 mph D) 1.6 mph North

1-4. What is the hiker from problem 1-3's average speed in miles per hour (mph)?

A) 1.6 mph North B) 3.2 mph C) 8 mph D) 4 mph

1-5. A ball is dropped off of a bridge. It hits the water below 3 seconds later. Ignoring air resistance, with what speed does the ball hit the water?

A) 9.8 m/s^2 B) 29.4 m/s C) 44.1m D) 44.1 m/s

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Problem 2 (25 points): The positions (in meters) of two objects, 1 and 2, are given by:

$$x_1(t) = 3\sin\left(\frac{\pi t}{4}\right) \quad , \quad x_2(t) = 3t^2 - 4t + 2$$

(a) (9 pts) What are the positions of the two objects at t=0 , 1 and 2s?

(b) (6 pts) Determine the acceleration and velocity of each object as functions of time.

(c) (10 pts) Give the average velocities of the objects between t=0s and t=2s, and instantaneous velocities of the objects at t= 2s.

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Problem 3 (25 points): A car is traveling along a flat, straight highway at a speed of 25 m/s. At the same time, a plane is flying 100m overhead in a straight line parallel to the car's path at a speed of 60 m/s. At the moment the plane is directly overhead, the passenger in the car shoots a ball up towards the plane at some angle θ.

a) (9pts) What initial speed and angle (with respect to the ground) are required for the ball to be caught by the pilot of the plane at the top of the ball's arc?

b) (6 pts) Give the position and velocity vectors for the car, the plane, and the ball as a function of time.

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Problem 3 Cont.

c) (10pts). Unfortunately, the pilot of the plane is not nearly coordinated enough to catch the ball. The pilot totally misses the ball and it continues on its trajectory. Where does the ball land? How long is the ball in the air? How far from the ball are the car and the plane when it lands on the ground?

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Problem 4 (25 points): A large box of mass 50kg lies on a frictionless inclined plane at an angle of 30º. Two movers attach a rope to the box so that they can pull on the box up the inclined plane (parallel to the surface of the plane).

a) (10 pts) Draw a diagram of the problem and draw a free body diagram for the block, showing all the Forces acting on the block. Don't forget to indicate your coordinate system.

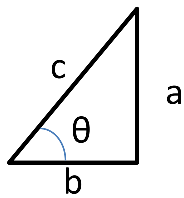
b) (5 pts) What is the Normal Force acting on the box?

c) (5 pts) If the box is initially at rest, with what Force do the movers need to pull on the box in order to keep it from sliding down the plane?

d) (5pts) The maximum force the movers can exert is 400 N. Give the equation of motion of the box if the movers are pulling with all of their might (assume box starts at rest).

95.141 Spring 2010: Exam I Formula Sheet

- Trig



$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$$c^2 = a^2 + b^2$$

- Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Misc. Formulas

Circumference of a circle: $2\pi r$

Area of a Circle: πr^2

Surface Area of a Sphere: $4\pi r^2$

Volume of a Sphere: $\frac{4}{3}\pi r^3$

Volume of a Cylinder: $h\pi r^2$

- Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \ (n \neq 0)$$

$$\frac{d}{dx}(\cos ax) = -a\sin ax \text{ (ax in radians)}$$

$$\frac{d}{dx}(\sin ax) = a\cos ax \text{ (ax in radians)}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad , \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\text{Chain Rule}: \frac{d}{dx}\left(f(g(x))\right) = \frac{df(g)}{dg}\frac{dg(x)}{dx}$$

$$\text{Distributive Rule}: \frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

- Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

- Conversions

$1\text{ liter} = 1\times10^{-3} m^3$, $1\text{ ft} = 0.3048 m$

$1\text{ mile} = 1609 m$, $1\text{ mile per hour} = 0.447\ ^{m}\!/_{s}$

$1 mm = 0.001 m$, $1 g = 0.001 kg$

- Units

Velocity $\rightarrow\ ^{dx}\!/_{dt} = \ ^{m}\!/_{s}$

Acceleration $\rightarrow\ ^{d^2x}\!/_{dt^2} = \ ^{m}\!/_{s^2}$

Force $\rightarrow$ Newton (N) $= \ ^{kgm}\!/_{s^2}$

Momentum $\rightarrow\ ^{kgm}\!/_{s}$

Torque $\rightarrow$ N - m

Angular Momentum $\rightarrow\ ^{kgm^2}\!/_{s}$

SI Units[mass, length, time] = [kg, m, s]

- Constants

$G = 6.67\times10^{-11}\ \frac{m^3}{kgs^2}$

$g = 9.8\ \frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$RadiusEarth = 6.4\times10^6\ m$

$MassEarth = 5.98\times10^{24}\ kg$

95.141 Spring 2010 : Formula Sheet

- One Dimensional Motion

$$\text{displacement} = \Delta x$$

$$\text{average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v(t) = \frac{dx(t)}{dt} \text{ (instantaneous)}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ (instantaneous)}$$

- Motion with constant a (one dimensional)

$$x(t) = x_o + v_{xo}t + \frac{1}{2}at^2$$

$$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$$

$$v^2 = v_{xo}^2 + 2a(x - x_o)$$

- Projectile Motion

For motion over level ground :

$$Range = \frac{v_o^2 \sin(2\theta_o)}{g}$$

- X-Y Plane Motion (with constant acceleration)

$$x(t) = x_o + v_{xo}t + \tfrac{1}{2}a_x t^2$$

$$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$$

$$y(t) = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$$

$$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$$

- Newton's Laws

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \quad , \sum F_y = ma_y \quad , \sum F_z = ma_z$$

$$if \quad \sum \vec{F} = 0 \quad then \quad \frac{d\vec{v}}{dt} = \vec{a} = 0$$