95.141 Exam 1a, February 24, 2010

Section Number _____

Section Instructor_____

Name _____,_____
　　　　　　　　Last Name　　　　　　　　First Name

Last 3 Digits of Student ID number: ____ ____ ____

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers! Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. **You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.**

Score on each problem:

1. _____

2. _____

3. _____

4. _____

Total Score (based on 100 pts)　　_____

Be prepared to show your student ID Card

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Problem 1 (25 points – 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

1-1.   A car travels at a constant velocity for 30s. During that time, the car travels a distance of 600m. What is the speed of the car?

A) 2 m/s   B) 18,000 m/s   C) 20m   D) 20m/s

1-2.   James Bond is running from a bad guy on the roof of a train which is moving with a speed of +45 m/s. If an observer on the ground sees Bond moving past her at a speed of +52 m/s, how fast is Bond running with respect to the roof of the train?

$v_{Bond}=$_____

1-3.   A 8kg mass is pushed across a frictionless horizontal surface with a horizontal force F=36N. What is the acceleration of the mass?

A) 9.8 m/s^2 B) 288 m/s^2 C) 4.5m/s^2 D) 288 m/s

1-4.   Suppose you have three vectors $\vec{V_1} = 2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{V_2} = 2\hat{i} - 4\hat{k}$, and $\vec{V_3} = -3\hat{i} - 6\hat{j} + 2\hat{k}$. What is $\vec{V} = \vec{V_1} - \vec{V_2} + \vec{V_3}$?

$$\vec{V} = \underline{\hspace{1cm}} \hat{i} + \underline{\hspace{1cm}} \hat{j} + \underline{\hspace{1cm}} \hat{k}$$

1-5.   An (incorrect) expression gives velocity v [m/s] in terms of mass m [kg], acceleration a [m/s^2], and position x [m], and a constant D. What are the units of D required to make the equation dimensionally correct?

$$v = \frac{Da}{mx^2}$$

$$[D] = \underline{\hspace{2cm}}$$

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Problem 2 (25 points): Two cars, starting from rest at x=0 at t=0, are racing. Car A and B both accelerate in a straight line at a constant acceleration (a=4m/s²) for 10s. After 10s Car A stops accelerating (acceleration is 0m/s²), but Car B continues to accelerate at the same rate (a=4m/s²).

(a) (5 pts) What are the cars' speed at t=10s?

$$v_{A,B}(t=10s)=\underline{\hspace{3cm}}$$

(b) (5 pts) How far have the cars traveled at t=10s?

$$x_{A,B}(t=10s)=\underline{\hspace{3cm}}$$

(c) (5 pts) After 20s, Car B crosses the finish line. How far is the finish line from the starting point of the race?

D=_____

(d) (10 pts) How far behind is Car A when Car B crosses the finish line?

Distance Behind=_____

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Problem 3 (25 points):   You launch a rock from the origin (0,0) with an initial speed of 50 m/s at an angle of $\theta_o=30°$ from horizontal.   Ignore air resistance, and assume the ground is perfectly flat.

a) (10pts)   Draw a coordinate system for this problem and show the trajectory of the rock. Give the initial velocity vector of the rock in component form.

$$\vec{v}_o = \underline{\hspace{1cm}} \hat{i} + \underline{\hspace{1.5cm}} \hat{j}$$

b) (8pts) How high does the rock go and how long does it take the rock to get to that height?

$$H=\underline{\hspace{1.5cm}} , \quad t_H= \underline{\hspace{1.5cm}}$$

c) (7pts) How far, horizontally, will the rock travel before it lands?

distance =_____

Problem 4 (25 points):   Two masses (m_a =3kg and m_b=5kg), initially at rest, are connected by a cord which passes over a massless pulley, as shown in the figure below.   The table and pulley system are assumed to be frictionless. A stop sits at a distance d=2m from mass b.
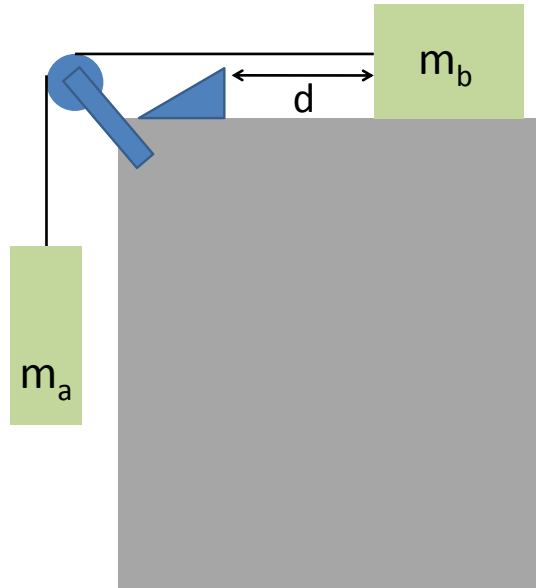
a) (10 pts) Draw free body diagrams for each of the blocks, showing all the Forces acting on the blocks. Don't forget to indicate your coordinate system for each block.

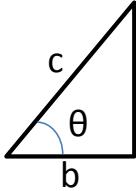b)   (5 pts) What is the acceleration of the blocks?

c)   (10pts) At a time t=0.5s from the time the blocks begin to move (t=0s), the cord breaks.At what time does mass b hit the stop?

95.141 Spring 2010: Exam I Formula Sheet

- ## Trig

$$\sin\theta = \frac{a}{c} \quad , \quad \cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

$$c^2 = a^2 + b^2$$

- ## Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- ## Misc. Formulas

Circumference of a circle: $2\pi r$

Area of a Circle: πr^2

Surface Area of a Sphere: $4\pi r^2$

Volume of a Sphere: $\frac{4}{3}\pi r^3$

Volume of a Cylinder: $h\pi r^2$

- ## Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \; (n \neq 0)$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \;(ax \text{ in radians})$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \;(ax \text{ in radians})$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x} \quad , \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

Chain Rule: $\dfrac{d}{dx}\left(f(g(x))\right) = \dfrac{df(g)}{dg}\dfrac{dg(x)}{dx}$

Distributive Rule: $\dfrac{d}{dx}\left(f(x) + g(x)\right) = \dfrac{df(x)}{dx} + \dfrac{dg(x)}{dx}$

- ## Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

- ## Conversions

$1\,\text{liter} = 1\times10^{-3}\,m^3 \quad , \quad 1\,\text{ft} = 0.3048m$

$1\,\text{mile} = 1609m \quad , \quad 1\,\text{mile per hour} = 0.447\,{}^m\!/_s$

$1mm = 0.001m \quad , \quad 1g = 0.001kg$

- ## Units

Velocity $\rightarrow {}^{dx}\!/_{dt} = {}^m\!/_s$

Acceleration $\rightarrow {}^{d^2x}\!/_{dt^2} = {}^m\!/_{s^2}$

Force $\rightarrow$ Newton$(N) = {}^{kgm}\!/_{s^2}$

Momentum $\rightarrow {}^{kgm}\!/_s$

Torque $\rightarrow$ N-m

Angular Momentum $\rightarrow {}^{kgm^2}\!/_s$

SI Units[mass, length, time] = [kg, m, s]

- ## Constants

$G = 6.67\times10^{-11}\,\frac{m^3}{kgs^2}$

$g = 9.8\,\frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$Radius\,Earth = 6.4\times10^6\,m$

$Mass\,Earth = 5.98\times10^{24}\,kg$

95.141 Spring 2010 : Formula Sheet

- ## One Dimensional Motion

$\text{displacement} = \Delta x$

$\text{average velocity} = \bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1}$

$\text{average acceleration} = \bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{v_2 - v_1}{t_2 - t_1}$

$v(t) = \dfrac{dx(t)}{dt} \text{ (instantaneous)}$

$a(t) = \dfrac{dv(t)}{dt} = \dfrac{d^2 x(t)}{dt^2} \text{ (instantaneous)}$

- ## Motion with constant a (one dimensional)

$x(t) = x_o + v_{xo}t + \dfrac{1}{2}at^2$

$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$

$v^2 = v_{xo}^2 + 2a(x - x_o)$

- ## Projectile Motion

For motion over level ground :

$Range = \dfrac{v_o^2 \sin(2\theta_o)}{g}$

- ## X-Y Plane Motion (with constant acceleration)

$x(t) = x_o + v_{xo}t + \tfrac{1}{2}a_x t^2$

$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$

$y(t) = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$

$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$

- ## Newton's Laws

$\sum \vec{F} = m\vec{a}$

$\sum F_x = ma_x \quad , \sum F_y = ma_y \quad , \sum F_z = ma_z$

$if \quad \sum \vec{F} = 0 \quad then \quad \dfrac{d\vec{v}}{dt} = \vec{a} = 0$