95.141 Exam 3a, November 25, 2009

Section Number $\qquad$
Section Instructor $\qquad$

Name $\qquad$ , Last Name First Name

Last 3 Digits of Student ID number: $\qquad$
Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers! Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.

## Score on each problem:

$\qquad$
2. $\qquad$
3. $\qquad$

## Total Score (based on 100 pts)

Be prepared to show your student ID Card

Name (last name only) $\qquad$
$\qquad$
Problem 1 (30 points - 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

For problems 1-1, 1-2, and 1-3: A merry-go-round ride starts at rest and accelerates with constant $\alpha=0.06 \mathrm{rad} / \mathrm{s}^{2}$ for 10 s , after which it moves with a constant angular velocity ( $\alpha=0 \mathrm{rad} / \mathrm{s}^{2}$ for $\mathrm{t}>10 \mathrm{~s}$ ).

1-1. What is the angular velocity of the merry-go-round at $\mathrm{t}=6 \mathrm{~s}$ ?
A) $0.6 \mathrm{rad} / \mathrm{s}$
B) $0.36 \mathrm{rad} / \mathrm{s}$
C) $0 \mathrm{rad} / \mathrm{s}$
D) $3 \mathrm{rad} / \mathrm{s}$

1-2. How far (in radians) has the merry-go-round rotated after 12s?
A) 4.2 rad
B) 3 rad
C) 7.2 rad
D) 0.3 rad

1-3. What is the tangential velocity at $\mathrm{t}=5 \mathrm{~s}$ of a child sitting 5 m from the axis of rotation of the merry-go-round?
A) $1.5 \mathrm{rad} / \mathrm{s}$
B) $3 \mathrm{~m} / \mathrm{s}$
C) $0.3 \mathrm{rad} / \mathrm{s}$
D) $1.5 \mathrm{~m} / \mathrm{s}$

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Problem 1 Cont.
1-4. What is the escape velocity for an object on the Moon?

$$
R_{\text {moon }}=1.74 \times 10^{6} \mathrm{~m} \quad, \quad M_{\text {moon }}=7.35 \times 10^{22} \mathrm{~kg} \quad, \quad G=6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}
$$

A) $1.799 \mathrm{~m} / \mathrm{s}$
B) $1,678 \mathrm{~m} / \mathrm{s}$
C) $2,374 \mathrm{~m} / \mathrm{s}$
D) $3235 \mathrm{~m} / \mathrm{s}$
$1-5$. A 1000 kg car climbs a hill at a constant speed of ( $\mathrm{v}=30 \mathrm{~m} / \mathrm{s}$ ). If the hill has a slope of $5^{\circ}$, and there is a drag force given by $F_{D}=0.4 v^{2}$, how much power must the car's engine put out?
A) 25.6 kW
B) 30.4 kW
C) 10.5 kW
D) 36.4 kW

1-6. A mass (m) is tied to a massless cord of length $r$, and dropped from rest at $\theta=90^{\circ}$. Give an expression for the speed of the mass as a function of $\theta$.

A) $v=\sqrt{g r \sin \theta}$
B) $v=\sqrt{2 g r \cos \theta}$
C) $v=\sqrt{2 g r}$
D) $v=\sqrt{2 g r(1-\cos \theta)}$

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Problem 2 ( 35 points): A projectile of mass $\mathrm{m}_{\mathrm{A}}=20 \mathrm{~g}$ is shot towards a block of mass $M_{B}=5 \mathrm{~kg}$ with a velocity of $\mathrm{v}_{\mathrm{A}}=400 \mathrm{~m} / \mathrm{s}$. The block is at rest, attached to a horizontal massless spring (at equilibrium) with $\mathrm{k}=500 \mathrm{~N} / \mathrm{m}$. :

(a) (6 pts) If the collision between the projectile and the mass is perfectly elastic, what are the velocities of the two masses immediately following the collision?
(b) (6 pts) For the collision in (a), how far does the spring compress, assuming the surface to be frictionless?
(c) $(5 \mathrm{pts})$ For the collision in (a), if the projectile is in contact with the mass for 0.5 ms , what is the average force exerted on the block by the projectile during the collision?

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Problem 2, continued
d) (8pts) Now, assume the collision is perfectly inelastic and the surface has a coefficient of kinetic friction of $\mu_{\mathrm{k}}=0.15$. What is the velocity of the projectile and block following the perfectly inelastic collision?
e) (10pts) For the situation described in (d), how far does the spring compress?

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Problem 3 ( 35 points): Imagine you have a cue ball and three billiard balls, of equal mass, arranged as shown in the diagram below.

a) (15pts) Determine the center of mass of this system.
b) (10pts) The cue ball is then shot at ball A with a speed of $3 \mathrm{~m} / \mathrm{s}$. Assuming the balls to be point masses, calculate the center of mass of the system at the instant the balls collide.

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Problem 3 Cont.
c) (10pts) The two balls collide elastically, and ball A leaves the collision with a velocity entirely in the positive y direction. What is the final speed and direction of both the cue ball and ball A?

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Problem 4(a) ( 35 points): A mother pushes her two children on a 200kg Merry-go-round of radius $\mathrm{R}=6 \mathrm{~m}$. Child $1\left(\mathrm{~m}_{1}=25 \mathrm{~kg}\right)$ sits in a seat a distance of $\mathrm{r}_{1}=3 \mathrm{~m}$ from the axis of rotation. Child $2\left(m_{2}=35 \mathrm{~kg}\right)$ sits in a seat a distance of $r_{2}=5 \mathrm{~m}$ from the axis of rotation. The mother pushes with a maximum force of 300 N at the edge of the merry-go-round. Assume the merry-go-round to be a uniform circle ( $I_{\text {circle }}=\frac{1}{2} M R^{2}$ ), and treat the children as point masses..

a) (10pts) Determine the center of mass of this system (children + merry-go-round) at the instant shown in the diagram above. Assume the origin $(0,0)$ is at the axis of rotation.
b) (10pts) Determine the moment of inertia of the system.

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Problem 4 Cont.
c) (5pts) What is the angular acceleration of the merry go round, assuming no friction?
d) (5pts) What is the angular acceleration of the merry-go-round if you include a frictional torque of $300 \mathrm{~N}-\mathrm{m}$ ?
e) (5pts) The outermost child falls off the merry-go-round at an angular velocity of $5 \mathrm{rad} / \mathrm{s}$, and the mother stops pushing to help the child. Assuming the same frictional torque from part (d), how long does it take the merry-go-round to stop turning?

