95.141 Exam 3a, May 5, 2010

Section Number ______________

Section Instructor_____________

Name ___________________________,____________________
Last Name First Name

Last 3 Digits of Student ID number: ____ ____ ____

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers! Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. **You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.**

Score on each problem:

1. _______

2. _______

3. _______

4. _______

Total Score (based on 100 pts) ________

Be prepared to show your student ID Card

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Problem 1 (25 points – 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

1-1. A figure skater starts a spin with her arms outstretched. Halfway through the spin, she brings her arms in, to her sides. This action will result in her angular momentum

A) Increasing B) Decreasing C) Remaining the Same D) Disappearing

1-2. Two objects sliding on a frictionless plane collide. What can you say, with certainty, about this collision?

A) Momentum is conserved B) It is perfectly inelastic

C) Kinetic Energy is Conserved D) Mechanical Energy is conserved

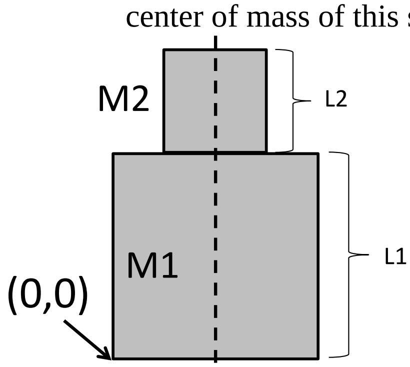
1-3. A disc accelerates from rest with constant α=0.8rad/s^2. During its angular acceleration it completes 5 revolutions. What is the disc's angular velocity after 5 revolutions?

A) 2.83 rad/s B) 7.09 rad/s C) 8.00 rad/s D) 50.26 rad/s

1-4. Calculate the vector cross product of $\vec{A} = 2\hat{i} + 8\hat{j}$ and $\vec{B} = -2\hat{j} + 4\hat{k}$.

$\vec{A} \times \vec{B} =$ ___________________

1-5. Two uniform 2D squares are stacked, centered, as shown below. What is the center of mass of this system? L1=2m, L2=1m, M1=4kg, M2=2kg.



CM=___________________

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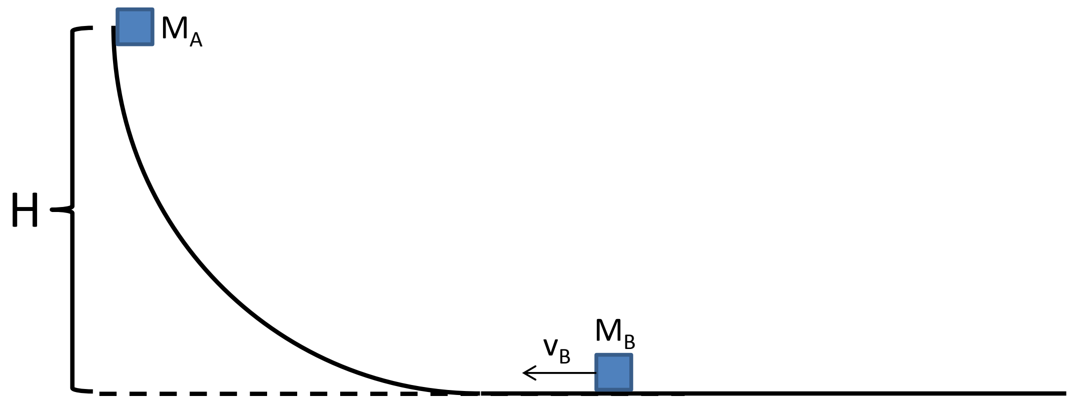
Problem 2 (25 points): A mass (M_A=3kg) slides down a frictionless ramp of height H=12m and collides with a second mass (M_B=5kg), moving towards mass A at v_B=8m/s . Assume the collision occurs with both masses moving horizontally.



a) (5 pts) What is the velocity of Mass A at the bottom of the ramp?

v_A =_____________

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b) (10pts) Assuming the collision is elastic, what are the velocities of the two masses after the collision?

v'_A =______________

v'_B =______________

c) (10pts) How far back up the ramp does Mass A travel?

H_{new}=______________

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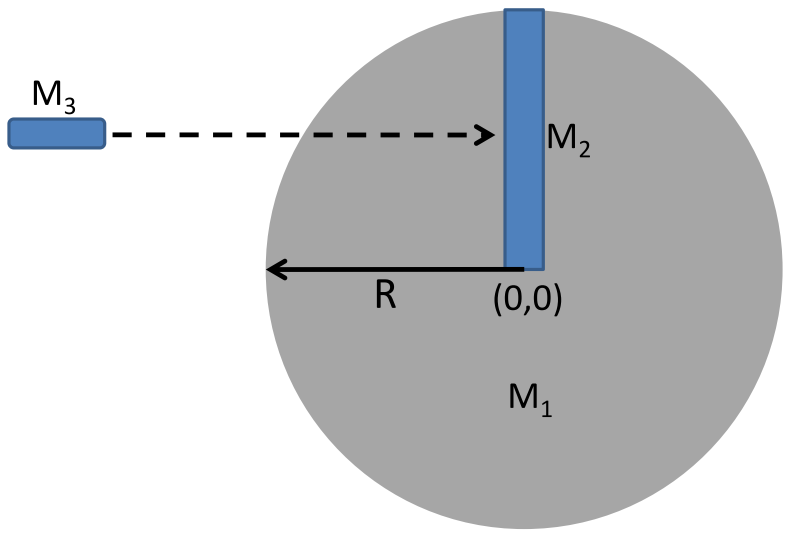
Problem 3 (25 points): A rectangular block (M2=12kg) is attached to a merry-go-round (M1=80kg, R=6m) such that the block extends from the axis of rotation to the edge of the merry-go-round. The moment of inertia of a disc is $I_{disc}=MR^2/2$, while the moment of inertia for a bar of length L rotating about one end is $I_{rect.}= ML^2/3$.



a) (5 pts) What is the moment of inertia (about the origin) of the merry-go-round and block?

$I_{Total}=$________________

b) (8 pts) A bullet (M_3=1kg) is shot with a velocity of v=300m/s into the midpoint of the rectangular block. Calculate the angular momentum of the bullet (about the origin) before it hits the block.

$L_{bullet}=$________________

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c) (5 pts) The bullet embeds in the block. What is the moment of inertia of the bullet + block + merry-go-round system after impact?

I' =_______________

d) (7pts) What is the angular velocity of the system following impact?

ω =_______________

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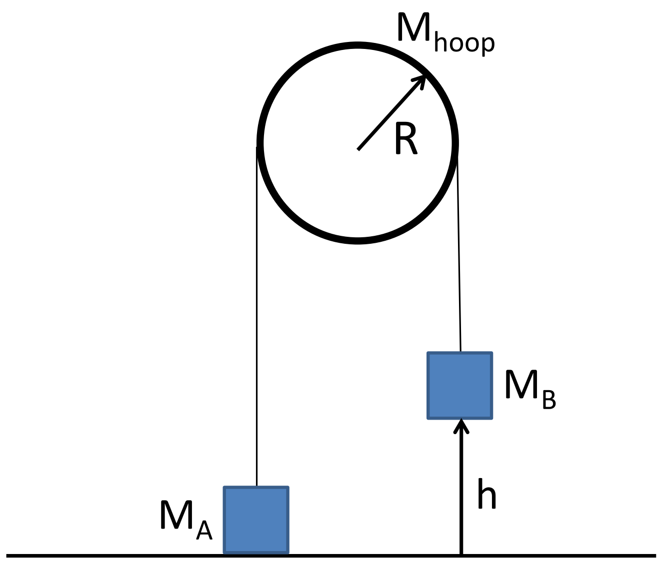
Problem 4 (25 points): Two mass (M_A=2kg, M_B=6kg) are connected by a massless cord wrapped around a hoop-shaped pulley of mass M=45kg. $I_{hoop}=MR^2$.



a) (5 pts) Assuming mass A is sitting just above the ground (no normal force), draw free body diagrams for two mass and the hoop. Indicate your coordinate system for each.

b) (5pts) Write out Newton's Second Law (in rotational or translational form) for each of the three objects.

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c) (10 pts) If the system is released from rest, what will the acceleration of the masses be?

a =_______________

d) (5pts) If mass B starts h=5m above the ground, how long will it take mass B to hit the ground?

t =_______________

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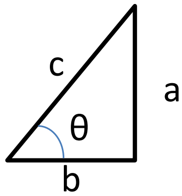
- Trig



$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$$c^2 = a^2 + b^2$$

- Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Misc. Formulas

Circumference of a circle : $2\pi r$

Area of a Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Volume of a Cylinder : $\mathrm{h}\pi r^2$

- Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \; (n \neq 0)$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x} \quad , \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\text{Chain Rule} : \frac{d}{dx}\left(f(g(x))\right) = \frac{df(g)}{dg}\frac{dg(x)}{dx}$$

$$\text{Distributive Rule} : \frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

- Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

- Conversions

$1\,\text{liter} = 1\times10^{-3} m^3$, $1\,\text{ft} = 0.3048m$

$1\,\text{mile} = 1609m$, $1\,\text{mile per hour} = 0.447\, {}^{m}\!/_{s}$

$1mm = 0.001m$, $1g = 0.001kg$

- Units

Velocity $\rightarrow {}^{dx}\!/_{dt} = {}^{m}\!/_{s}$

Acceleration $\rightarrow {}^{d^2x}\!/_{dt^2} = {}^{m}\!/_{s^2}$

Force $\rightarrow$ Newton (N) $= {}^{kgm}\!/_{s^2}$

Momentum $\rightarrow {}^{kgm}\!/_{s}$

Torque $\rightarrow$ N - m

Angular Momentum $\rightarrow {}^{kgm^2}\!/_{s}$

SI Units [mass, length, time] = [kg, m, s]

- Constants

$G = 6.67\times10^{-11} \frac{m^3}{kgs^2}$

$g = 9.8 \frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$RadiusEarth = 6.4\times10^6 \; m$

$MassEarth = 5.98\times10^{24} \; kg$

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- One Dimensional Motion

$$\text{displacement} = \Delta x$$

$$\text{average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v(t) = \frac{dx(t)}{dt} \text{ (instantaneous)}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ (instantaneous)}$$

- Motion with constant a (one dimensional)

$$x(t) = x_o + v_{xo}t + \frac{1}{2}at^2$$

$$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$$

$$v^2 = v_{xo}^2 + 2a(x - x_o)$$

- Projectile Motion

For motion over level ground :

$$Range = \frac{v_o^2 \sin(2\theta_o)}{g}$$

- X-Y Plane Motion (with constant acceleration)

$$x(t) = x_o + v_{xo}t + \tfrac{1}{2}a_x t^2$$

$$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$$

$$y(t) = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$$

$$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$$

- Circular Motion

$$\text{Arc Length} = R\Delta\theta$$

$$\omega = \frac{d\theta}{dt} \quad , \quad v_{\tan} = \omega R$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad \omega = 2\pi f$$

$$a_{centripetal} = \frac{v^2}{R} = R\omega^2$$

- Dot, Cross Product

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \bullet \vec{B} = (A_xB_x + A_yB_y + A_zB_z) = |A||B|\cos\theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$= |A||B|\sin\theta$$

- Frictional Forces

$$F_{s-\max} = \mu_s F_N \quad , \quad F_k = \mu_k F_N \quad , \quad \mu_s > \mu_k$$

- Work and Energy

$$\text{Work Energy Theorem} \rightarrow \mathrm{W}_{net} = \Delta KE$$

$$KE = \frac{1}{2}mv^2 \quad , \quad \Delta KE = KE_2 - KE_1$$

$$\text{One Dimension, F constant: W} = \mathrm{F}\Delta\mathrm{x}$$

$$\text{More Generally : W} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{\mathrm{F}} \bullet \mathrm{d}\vec{r}$$

$$\text{Power} : P = \frac{dW}{dt} \text{ (Watts)}$$

$$\text{1-D} : P = Fv$$

- Potential Energy

$$U(x) = U_o(x_o) - \int_{x_o}^{x} F dx \quad , \quad F(x) = -\frac{dU(x)}{dx}$$

$$\text{Gravity of Earth's Surface : } \mathrm{U(y)} = \mathrm{U_o} + \mathrm{mg}\Delta\mathrm{y}$$

General Expression for Gravitational Potential:

$$U(r) = -\frac{GM_1M_2}{r}$$

$$\text{For Springs : } U(x) = \frac{1}{2}kx^2$$

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- Conservative Energy Systems

$$E_{tot} = K + U \text{ (constant)}$$

- Center of Mass

$$\vec{r}_{CM} = \frac{1}{M_{total}}\left[m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...m_n\vec{r}_n\right] = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

$$\text{for 1D}: x_{cm} = \frac{1}{M_{total}}\left[m_1x_1 + m_2x_2 + m_3x_3 + ...m_nx_n\right]$$

$$\vec{F}_{net} = M_{total}\vec{a}_{CM}$$

- Momentum and Impulse

$$\vec{p} = m\vec{v} \quad , \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{Impulse}: \vec{J} = \int \vec{F}dt = F_{ave}\Delta t = \Delta\vec{p}$$

If no external Forces (only internal Forces) then momentum is conserved.

- 1-D Elastic Collisions (2masses)

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \text{ KE, } \vec{\text{p}} \text{ conserved}$$

- Rotational Motion (const. α)

$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega_o + \alpha t$$

$$\alpha(t) = \alpha \text{ (constant)}$$

- Rotational Motion (general)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad KE_{rot} = \frac{1}{2}I\omega^2$$

$$I = \sum_i m_i R_i^2 = \int \vec{r}^{\,2}$$

$$\text{Parallel Axis Theorem}: I_{\parallel} = I_{CM} + Mh^2$$

$$\vec{\tau} = RF\sin\theta = \vec{R}\times\vec{F} = I\vec{\alpha}$$

$$\vec{L} = I\vec{\omega} = \vec{R}\times\vec{p}$$

$$v_{\tan} = R\omega \quad , \quad a_{\tan} = R\alpha$$

- Kepler's 3rd Law, Force Grav.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \quad , \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$F_{gravity} = \frac{-GM_1M_2}{r^2}$$

- Harmonic Motion

$$F = -kx \quad , \quad U(x) = \frac{1}{2}kx^2 \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos(\omega t + \phi)$$

Damped Harmonic Motion (damp. const. b)

$$x(t) = Ae^{-\gamma t}\cos(\omega' t + \phi)$$

$$\gamma = b/2m \quad , \quad \omega' = \sqrt{k/m - b^2/4m^2}$$

Forced Harmonic Motion (Forcing freq. ω)

$$x(t) = A(\omega)\cos(\omega t + \phi_o) \quad , \quad \omega_o = \sqrt{k/m}$$

$$A(\omega) = \frac{F_o}{m\sqrt{(\omega^2 - \omega_o^2)^2 + b^2\omega^2/m^2}}$$

- Pendulums $\quad \omega = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T}$

- Waves

λ = wavelength, distance between crests (fixed t)

T = Period, time between crests (fixed x)

Equation for wave travelling in + x direction:

$$D(x,t) = A\sin(kx - \omega t) \text{ or } A\cos(kx - \omega t)$$

Equation for wave travelling in − x direction:

$$D(x,t) = A\sin(kx + \omega t) \text{ or } A\cos(kx + \omega t)$$

$$\lambda = \frac{2\pi}{k}$$

$$v_{wave} = \lambda f \quad , \quad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\text{For Strings with tension T}: v = \sqrt{\frac{T}{\mu}}$$