95.141 Final Exam, December 21, 2009

Section Number _____________

Section Instructor____________

Name __________________________,____________________

Last Name First Name

Last 3 Digits of Student ID number: ____ ____ ____

Answer all questions, beginning each new question in the space provided. Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers! Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. **You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.**

Score on each problem:

1. _______ **5.** _______

2. _______ **6.** _______

3. _______ **7.** _______

4. _______ **8.** _______

Total Score (based on 200 pts) _______

Be prepared to show your student ID Card

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Problem 1 (25 points – 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

1-1. A centrifuge takes 5 seconds to accelerate from rest to 20,000 rpm. Assuming constant acceleration, what is the angular acceleration of the centrifuge?

A) 405 rad/s B) 419 rad/s^2 C) 2094 rad/s D) 4000 rad/s^2

1-2. What is the torque (in N-m) exerted by the Force $\vec{F}$ below, when exerted at $\vec{r}$?

$$\vec{r} = 2\hat{i} + 4\hat{k} \qquad \vec{F} = 4\hat{i} - 3\hat{j}$$

A) $12\hat{i} + 16\hat{j} - 6\hat{k}$ B) 0 C) $4\hat{i} - 6\hat{j} + 3\hat{k}$ D) -4

For problems 1-3, 1-4, 1-5, and 1-6:

$$D(x,t) = 3\sin(\frac{\pi}{4}x - 6\pi t)$$

1-3. What is the amplitude of the wave described by D(x,t)?

A= ___________

1-4. What is the wavelength of the wave described by D(x,t)?

λ= ______________

1-5. With what speed is the wave described by D(x,t) travelling?

v= ________________

1-6. (3pts Extra Credit) Is the wave described by D(x,t) travelling in the +x or –x direction?

A) +x B) -x

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Problem 2 (25 points): A large box of mass 80kg lies on a frictionless inclined plane at an angle of 25º. Two movers attach a rope to the box so that they can pull on the box up the inclined plane (parallel to the surface of the plane).

a) (10 pts) Draw a diagram of the problem and draw a free body diagram for the block, showing all the Forces acting on the block. Don't forget to indicate your coordinate system.

b) (5 pts) What is the Normal Force acting on the box?

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Problem 2 Cont.

c) (5 pts) If the box is initially at rest, with what combined Force do the movers need to pull on the box in order to keep it from sliding down the plane?

d) (5pts) The maximum combined force the movers can exert is 400 N. Give the equation of motion of the box, x(t), if the movers are pulling with all of their might (assume box starts at rest).

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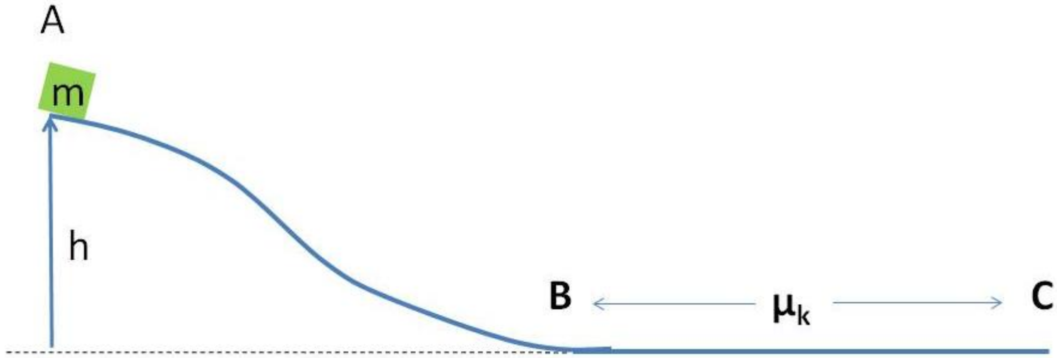
Problem 3 (20 points): A mass (5kg), released from rest, slides down a frictionless track of height h=10m (from point A to pointB) onto a flat surface (from point B on) with a coefficient of friction of μ_k=0.15. The mass comes to rest at point C.



a) (5 pts) Give speed of the mass at point B.

b) (15pts) How far is point C from point B? How much work is done by the Force of Friction on the mass over the entire length of the course?

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Problem 4 (25 points): A simple harmonic oscillator consists of a 4kg mass attached to a horizontal spring. The mass rests on a frictionless surface.

a) (10pts) The spring/mass system is compressed and released from rest. It is observed to oscillate with a period T=2s. What is the spring constant of the spring?

b) (5 pts) The spring/mass system is returned to equilibrium. It is then given an initial velocity of +7m/s and allowed to oscillate back and forth. What is the amplitude of oscillation of the spring/mass system?

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Problem 4 Cont.

c) (10pts) Give the position and velocity of the mass as a function of time ($x(t)$ and $v(t)$), for the intial conditions of part(b). Hint: Check your answers with the initial conditions given in part (b).

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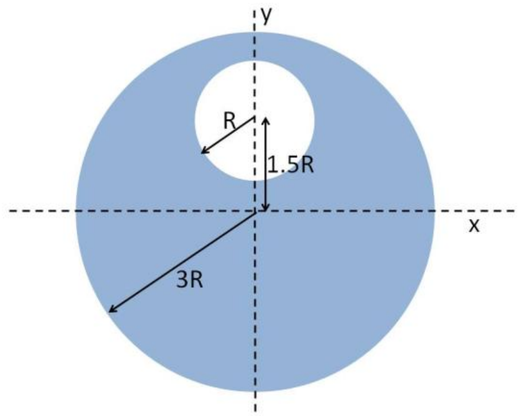
Problem 5 (20 points): A uniform circular plate of radius 3R has a hole of radius R cutout of it. The center of the hole is a distance 1.5R from the center of the larger circle and lies on the y axis. What is the position of the center of mass of the plate? (Hint: try subtraction).



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Problem 6 (20 points): Principal Skinner (70kg) wants to turn the container by 90° in 30s. He decides to do so by running in a circular path of radius R_s=1.25m (with constant speed) on the top of the container. Assume the container has a moment of inertia of 40,000kg-m^2 and is initially at rest before Skinner starts running.

a) (5 pts) What is the desired angular velocity (in rad/s) of the container?

b) (5pts) What is the moment of inertia of Principal Skinner, using the center of his circular path as the axis of rotation?

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Problem 6, continued.

c) (10pts) Using the principle of conservation of angular momentum, determine how fast Principal Skinner must run (in m/s) to move the container, with a constant angular velocity, 90° in 30s.

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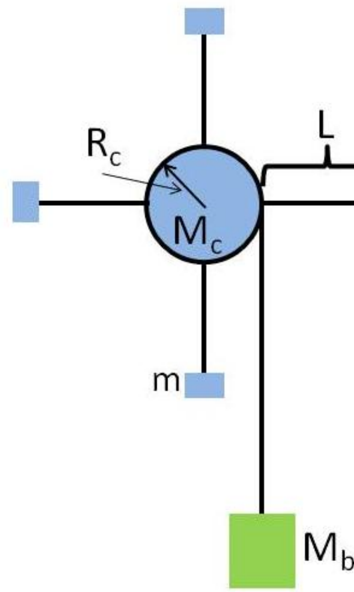
Problem 7 (35 points): The system below consists of a cylinder (R_c=20cm, M_c=60kg) and 4 point masses (m=3kg each) attached to the outer edge of the cylinder by massless rods of length L=0.25m each. The system is free to rotate about the axis through the center of the cylinder. The moment of inertia of a cylinder is given by: $I_{cyl.} = \frac{1}{2}MR^2$.

(a) (10 pts) What is the moment of inertia of this system for rotation about the center of the cylinder? (Do not count M_b as part of the system)



(a) (10 pts) Assume an 8kg mass (M_b) is attached to a massless cord, wrapped around the cylinder, and dropped from rest. What is the acceleration of the mass (M_b)?

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Problem 7, continued

(b) (5 pts) What is the angular acceleration of the rotating cylinder/4mass system?

(c) (10pts) Determine the Kinetic Energy, as a function of time, associated with the i) rotating system and ii) the falling mass.

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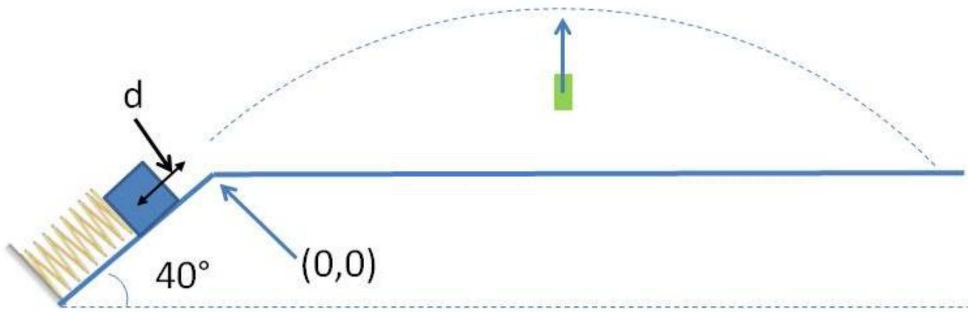
Problem 8 (30 points): A mass of 2kg on an angled spring (k=800 N/m) compressed by certain distance d from equilibrium. The spring/mass system is in equilibrium when the mass is at ground level (y=0) and the angled surface is frictionless. The spring is released, and the mass is shot into the air with an initial *speed* v_o=50m/s.



a) (5pts) What is the initial *velocity* of the mass when it leaves the spring at (0,0).

$$\vec{v}_o = \_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{i} + \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{j}$$

b) (10pts) How far from the launch point (0,0), is the mass at the peak of its trajectory (write in component form)?

$$\vec{r}_{peak} = \_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{i} + \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{j}$$

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Problem 8 Cont.

c) (10pts) At the peak of its trajectory, a 30g bullet, shot from directly below the mass embeds in the mass with a speed of 420m/s. What is the *velocity* of the mass/bullet immediately following the collision?

$$\vec{v}' = \_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{i} + \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \hat{j}$$

d) (5 pts) Give the initial compression (d) of the spring. Remember to take into account the gravitational potential energy of the mass.

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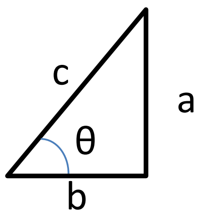
- Trig



$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$$c^2 = a^2 + b^2$$

- Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Misc. Formulas

Circumference of a circle : $2\pi r$

Area of a Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Volume of a Cylinder : $h\pi r^2$

- Derivatives

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ $(n \neq 0)$

$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax$ (ax in radians)

$\frac{d}{dx}\left(\sin ax\right) = a\cos ax$ (ax in radians)

$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$, $\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$

Chain Rule : $\frac{d}{dx}\left(f(g(x))\right) = \frac{df(g)}{dg}\frac{dg(x)}{dx}$

Distributive Rule : $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$

- Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

- Conversions

$1\,\text{liter} = 1\times10^{-3} m^3$, $1\,\text{ft} = 0.3048m$

$1\,\text{mile} = 1609m$, $1\,\text{mile per hour} = 0.447\, {}^{m}\!/_{s}$

$1mm = 0.001m$, $1g = 0.001kg$

- Units

Velocity $\rightarrow {}^{dx}\!/_{dt} = {}^{m}\!/_{s}$

Acceleration $\rightarrow {}^{d^2x}\!/_{dt^2} = {}^{m}\!/_{s^2}$

Force $\rightarrow$ Newton (N) $= {}^{kgm}\!/_{s^2}$

Momentum $\rightarrow {}^{kgm}\!/_{s}$

Torque $\rightarrow$ N - m

Angular Momentum $\rightarrow {}^{kgm^2}\!/_{s}$

SI Units[mass, length, time] = [kg, m, s]

- Constants

$G = 6.67\times10^{-11}\ \frac{m^3}{kgs^2}$

$g = 9.8\ \frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$RadiusEarth = 6.4\times10^6\ m$

$MassEarth = 5.98\times10^{24}\ kg$

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- One Dimensional Motion

$$\text{displacement} = \Delta x$$

$$\text{average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v(t) = \frac{dx(t)}{dt} \text{ (instantaneous)}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ (instantaneous)}$$

- Motion with constant a (one dimensional)

$$x(t) = x_o + v_{xo}t + \frac{1}{2}at^2$$

$$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$$

$$v^2 = v_{xo}^2 + 2a(x - x_o)$$

- Projectile Motion

For motion over level ground :

$$Range = \frac{v_o^2 \sin(2\theta_o)}{g}$$

- X-Y Plane Motion (with constant acceleration)

$$x(t) = x_o + v_{xo}t + \tfrac{1}{2}a_x t^2$$

$$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$$

$$y(t) = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$$

$$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$$

- Circular Motion

$$\text{Arc Length} = R\Delta\theta$$

$$\omega = \frac{d\theta}{dt} \quad , \quad v_{\tan} = \omega R$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad \omega = 2\pi f$$

$$a_{centripetal} = \frac{v^2}{R} = R\omega^2$$

- Dot, Cross Product

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\bullet\vec{B} = (A_xB_x + A_yB_y + A_zB_z) = |A||B|\cos\theta$$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$= |A||B|\sin\theta$$

- Frictional Forces

$$F_{s-\max} = \mu_s F_N \quad , \quad F_k = \mu_k F_N \quad , \quad \mu_s > \mu_k$$

- Work and Energy

$$\text{Work Energy Theorem} \rightarrow \mathrm{W}_{net} = \Delta KE$$

$$KE = \frac{1}{2}mv^2 \quad , \quad \Delta KE = KE_2 - KE_1$$

$$\text{One Dimension, F constant}: \mathrm{W} = \mathrm{F}\Delta \mathrm{x}$$

$$\text{More Generally}: \mathrm{W} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{\mathrm{F}}\bullet d\vec{r}$$

$$\text{Power}: P = \frac{dW}{dt} \text{ (Watts)}$$

$$\text{1-D}: P = Fv$$

- Potential Energy

$$U(x) = U_o(x_o) - \int_{x_o}^{x} F dx \quad , \quad F(x) = -\frac{dU(x)}{dx}$$

$$\text{Gravity of Earth's Surface}: \mathrm{U(y)} = \mathrm{U_o} + \mathrm{mg}\Delta \mathrm{y}$$

General Expression for Gravitational Potential:

$$U(r) = -\frac{GM_1M_2}{r}$$

$$\text{For Springs}: U(x) = \frac{1}{2}kx^2$$

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- Conservative Energy Systems

$$E_{tot} = K + U \text{ (constant)}$$

- Center of Mass

$$\vec{r}_{CM} = \frac{1}{M_{total}}\left[m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...m_n\vec{r}_n\right] = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

$$\text{for 1D}: x_{cm} = \frac{1}{M_{total}}\left[m_1x_1 + m_2x_2 + m_3x_3 + ...m_nx_n\right]$$

$$\vec{F}_{net} = M_{total}\vec{a}_{CM}$$

- Momentum and Impulse

$$\vec{p} = m\vec{v} \quad , \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{Impulse}: \vec{J} = \int \vec{F}dt = F_{ave}\Delta t = \Delta\vec{p}$$

If no external Forces (only internal Forces) then momentum is conserved.

- 1-D Elastic Collisions (2masses)

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \text{ KE, } \vec{\mathrm{p}} \text{ conserved}$$

- Rotational Motion (const. α)

$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega_o + \alpha t$$

$$\alpha(t) = \alpha \text{ (constant)}$$

- Rotational Motion (general)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad KE_{rot} = \frac{1}{2}I\omega^2$$

$$I = \sum_i m_i R_i^2 = \int \vec{r}^{\,2}$$

$$\text{Parallel Axis Theorem}: I_{\parallel} = I_{CM} + Mh^2$$

$$\vec{\tau} = RF\sin\theta = \vec{R}\times\vec{F} = I\vec{\alpha}$$

$$\vec{L} = I\vec{\omega} = \vec{R}\times\vec{p}$$

$$v_{\tan} = R\omega \quad , \quad a_{\tan} = R\alpha$$

- Kepler's 3rd Law, Force Grav.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \quad , \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$F_{gravity} = \frac{-GM_1M_2}{r^2}$$

- Harmonic Motion

$$F = \text{-}kx \quad , \quad U(x) = \frac{1}{2}kx^2 \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos(\omega t + \phi)$$

Damped Harmonic Motion (damp. const. b)

$$x(t) = Ae^{-\gamma t}\cos(\omega' t + \phi)$$

$$\gamma = b/2m \quad , \quad \omega' = \sqrt{k/m - b^2/4m^2}$$

Forced Harmonic Motion (Forcing freq. ω)

$$x(t) = A(\omega)\cos(\omega t + \phi_o) \quad , \quad \omega_o = \sqrt{k/m}$$

$$A(\omega) = \frac{F_o}{m\sqrt{(\omega^2 - \omega_o^2)^2 + b^2\omega^2/m^2}}$$

- Pendulums $\omega = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T}$

- Waves

λ = wavelength, distance between crests (fixed t)

T = Period, time between crests (fixed x)

Equation for wave travelling in + x direction:

$$D(x,t) = A\sin(kx - \omega t) \text{ or } A\cos(kx - \omega t)$$

Equation for wave travelling in – x direction:

$$D(x,t) = A\sin(kx + \omega t) \text{ or } A\cos(kx + \omega t)$$

$$\lambda = \frac{2\pi}{k}$$

$$v_{wave} = \lambda f \quad , \quad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\text{For Strings with tension T}: v = \sqrt{\frac{T}{\mu}}$$