95.141 Final Exam a, May 15, 2010

Section Number ______________

Section Instructor_____________

Name ___________________________,____________________

Last Name First Name

Last 3 Digits of Student ID number: ____ ____ ____

Answer all questions, beginning each new question in the space provided. <u>Show all work. Show all formulas used for each problem prior to substitution of numbers. Label diagrams and include appropriate units for your answers!</u> Write your name and section number at the top of each page in the space provided and write the name of your section instructor in the place provided on this cover sheet. **You may use an alphanumeric calculator (one which exhibits physical formulas) during the exam, as long as you do not program any numbers to memory. By using an alphanumeric calculator you agree to allow us to check its memory during the exam. Simple scientific calculators are always OK.**

Score on each problem:

1. ________ **5.** ________

2. ________ **6.** ________

3. ________ **7.** ________

4. ________ **8.** ________

Total Score (based on 200 pts) ________

<u>Be prepared to show your student ID Card</u>

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Problem 1 (25 points – 5 points each, no partial credit on this problem only, don't forget units if a fill-in answer)

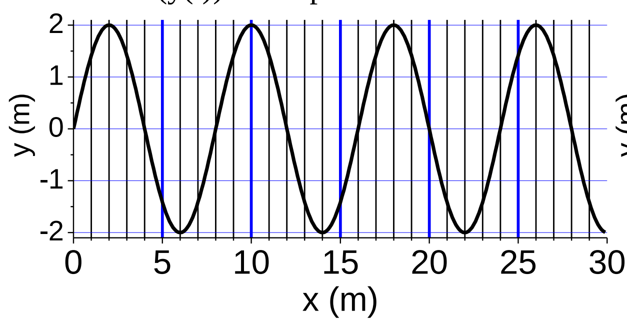
1-1. A mass of 2kg starts from rest at the origin and accelerates with $\vec{a} = 3\hat{i} - 2\hat{j} + 5\hat{k}$. Give the net Force acting on the mass.

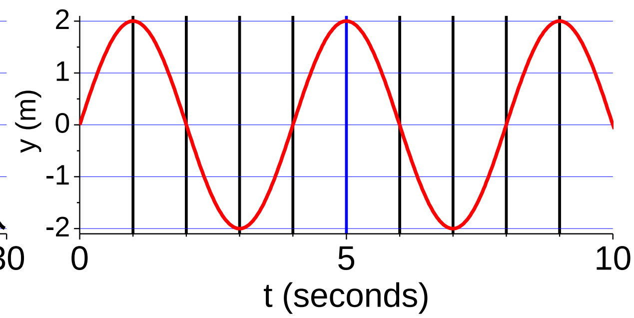
$\vec{F}$ =___________________________

1-2. What is the position of the mass from part (a) after 3s?

$\vec{r}$ =___________________________

For problems 1-3, 1-4, 1-5, and 1-6: The plots below show (left) the displacement of a travelling wave (y(x)) at t=0s, and (right) the displacement of the wave as a function of time (y(t)) at the point x=0.





1-3. What is the amplitude of the wave described by the plot above?

A= ___________

1-4. What is the wavelength of the wave described by the plot above?

λ= ______________

1-5. What is the Period of the wave described by the plots above?

T= ________________

1-6. (3pts Extra Credit) With what speed is the wave travelling?

v= ________________

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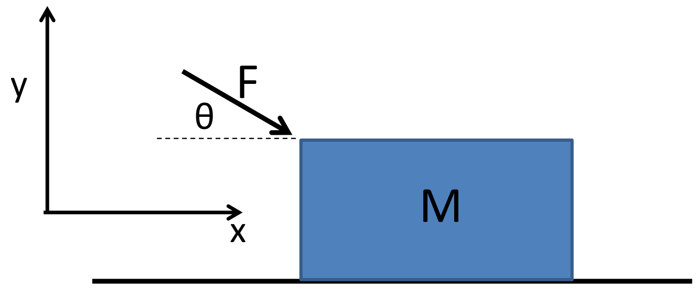
Problem 2 (25 points): A 6kg block sits on a surface with μ_s=0.5 and μ_k=0.3. You exert a force (F) at an angle of θ=30°on this block.



a) (5 pts) Draw the free body diagram for the block.

b) (10pts) What is the minimum Force with which you can push the block and have it move?

F_{min}=________________

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c) (5 pts) Suppose the magnitude of the force shown in the figure is 50N, and the block begins to move. What will the acceleration of the block be?

a=_______________

a) (5pts) You apply this Force (50N) for 8s. If the block starts from rest, what is the net work done on the block over this time?

W_{net}=_______________

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Problem 3 (25 points): A satellite (M=2500kg) orbits the earth at an altitude of 2000km. M_E=5.98x10^{24}kg, R_E=6.38x10^6m.

a) (6 pts) What is the magnitude of the gravitational force (from the Earth) acting on the satellite?

F_g=____________

b) (6 pts) What is the satellite's speed?

v_s=____________

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c) (7pts) What is the gravitational potential energy of the satellite? Remember, the gravitational potential should be 0 when $R=\infty$, and negative for any distance closer….

PE =_____________

d) (6pts) What is the total energy (kinetic + potential) of the satellite?

E =_____________

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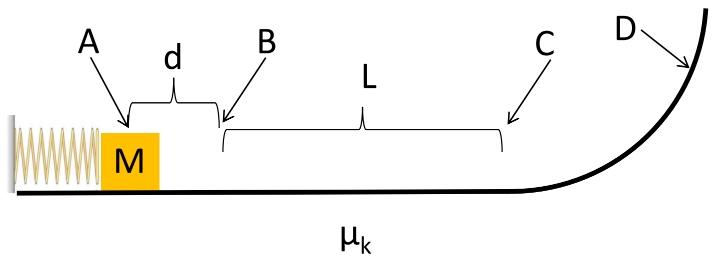
Problem 4 (25 points): A mass (M=2kg) is placed in front of a spring with k=900N/m, compressed d=50cm. The spring is released, shooting the mass forward from A to the spring's equilibrium position at B (A to B is frictionless). The mass then travels along a flat surface from B to C (L=20m), with μ_k=0.15. At C the surface becomes frictionless and smoothly inclines upwards. The speed of the mass at point D is 0m/s.



a) (5 pts) What is the velocity of the mass at B?

v_B =____________

b) (5 pts) What is the velocity of the mass at point C?

v_C =____________

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c) (5pts) How high is point D above the flat surface?

H_D =_______________

d) (10pts) Does the mass make it back to the spring? No credit for simple yes or no answer, must explain why or why not!

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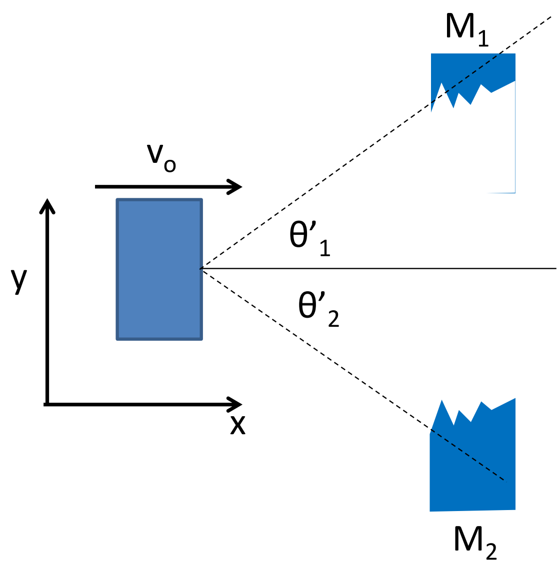
Problem 5 (25 points): A block of mass M=8kg slides across a frictionless surface with a speed of v_o=25 m/s, as shown below. A small explosive on the block is detonated remotely, and the block is broken into two pieces (M_1=2kg and M_2=6kg). M_1 leaves the explosion with a speed of 30m/s at an angle of $\theta_1' = 30°$.



a) (10 pts) What is the speed of M_2 after the explosion?

$v_2' =$ ____________

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b) (10 pts) At what angle does M_2 leave the collision?

$\theta_2' =$ ____________

c) (5 pts) What is the minimal energy of the explosion? Hint: How much did the kinetic energy of the system change by?

$E_{explosion} =$ ____________

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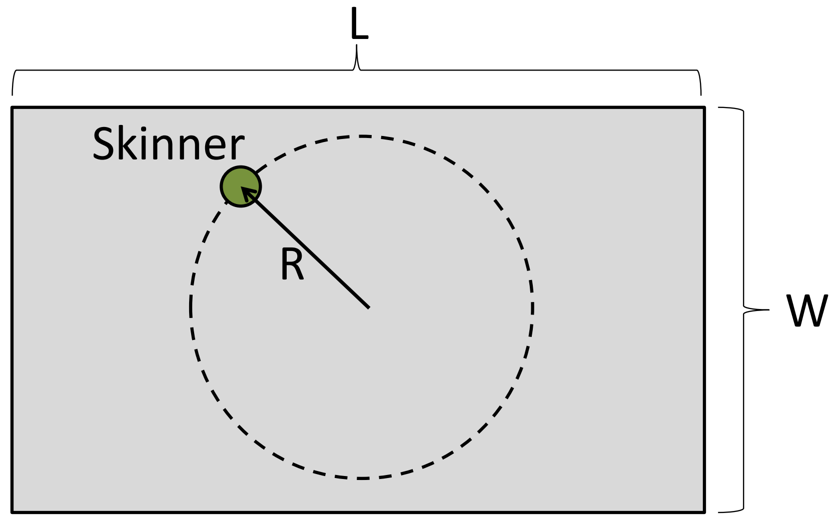
Problem 6 (25 points): A rectangular platform of mass Mp=3000kg, width W=5m and length L=10m is free to rotate around its center of mass. The platform needs to be rotated 60° in 30s. Principal Skinner (75kg) knows how to do this! He jumps on the container and begins "spazzing" out, running with constant speed in circles of radius R=2m around the platform's center of mass. $I_{rectangle}=M(L^2+W^2)/12$.



a) (5 pts) What is the minimum angular velocity (in rad/s) the container must have?

ω_{min} =______________

b) (5pts) What is the moment of inertia of Principal Skinner, using the center of his circular path as the axis of rotation?

$I_{skinner}$=______________

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c) (10pts) Using the principle of conservation of angular momentum, determine how fast Principal Skinner must run (in m/s) to move the container, with a constant angular velocity, 60° in 30s.

$V_{skinner}$=________________

d) (5pts) Suppose, instead of running at a constant speed, Skinner starts from rest and runs faster and faster (along the same circular path), with an angular acceleration of α=0.5rad/s^2. Determine whether Skinner will have moved the container 60° after 30 seconds. Must show work to receive any credit!!

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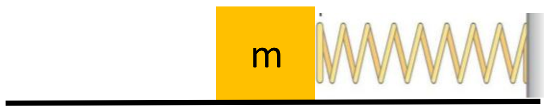
Problem 7 (25 points): A mass (m=5kg) is attached to a horizontal spring and placed on a frictionless surface. The spring-mass system is compressed, and released. The mass completes 5 full oscillations in 4 seconds.



(a) (6 pts) What is spring-mass system's period of oscillation and natural frequency?

T=_____________

ω=_____________

(b) (4 pts) What is the spring constant of the spring?

k=_____________

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Problem 7, continued

(c) (5 pts) The spring-mass system is stopped and placed at equilibrium. The system is then given an initial velocity of 5 m/s. What is the total energy of the spring-mass system?

E=_____________

(d) (5pts) What is the spring-mass system's amplitude of oscillation from part (c)?

A=_____________

(e) (5pts) Assume there exists some velocity-dependent air resistance Force on the harmonic oscillator described in (a)-(d). Plot the general form for x(t) for the harmonic oscillator with air resistance on the axes provided below.

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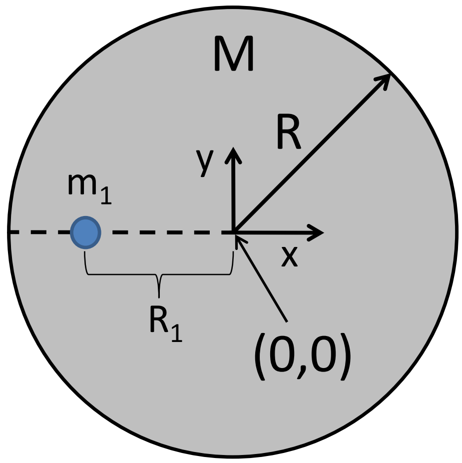
Problem 8 (25 points): The system below consists of a circular disc (M_d=100kg, R=6m) and a point mass (M_1=50kg) a distance R_1=4m from the origin in the –x direction. The point mass is attached to the disc, which lies on a frictionless surface. $I_{disc}=MR^2/2$.



(a) (5 pts) What is the center of mass of this system?

$(x,y)_{CM}$=______________

(b) (5 pts) What is the moment of inertia of the system *about its center of mass*? Hint: you will need to use the parallel axis theorem.

I_{CM}=______________

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Problem 8, continued

A Force $\vec{F} = 6\hat{j}$ is applied to the mass (m_1).

(c) (5 pts) What is the initial acceleration of the center of mass of the system?

$\vec{a}_{CM}$ = _______________

(d) (5pts) What is the initial torque about the center of mass from this Force?

$\vec{\tau}$ = _______________

(e) (5pts) What is the initial angular acceleration of this system about its center of mass?

$\vec{\alpha}$ = _______________

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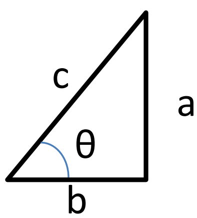
- Trig



$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$$c^2 = a^2 + b^2$$

- Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Misc. Formulas

Circumference of a circle : $2\pi r$

Area of a Circle : πr^2

Surface Area of a Sphere : $4\pi r^2$

Volume of a Sphere : $\frac{4}{3}\pi r^3$

Volume of a Cylinder : $h\pi r^2$

- Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1} \; (n \neq 0)$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x} \quad , \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

Chain Rule : $\frac{d}{dx}\left(f(g(x))\right) = \frac{df(g)}{dg}\frac{dg(x)}{dx}$

Distributive Rule : $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$

- Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

- Conversions

$1 \text{ liter} = 1\times10^{-3} m^3$, $1 \text{ ft} = 0.3048 m$

$1 \text{ mile} = 1609 m$, $1 \text{ mile per hour} = 0.447 \, m/s$

$1 mm = 0.001 m$, $1 g = 0.001 kg$

- Units

Velocity $\rightarrow dx/dt = m/s$

Acceleration $\rightarrow d^2x/dt^2 = m/s^2$

Force $\rightarrow$ Newton (N) $= kgm/s^2$

Momentum $\rightarrow kgm/s$

Torque $\rightarrow$ N - m

Angular Momentum $\rightarrow kgm^2/s$

SI Units[mass, length, time] = [kg, m, s]

- Constants

$G = 6.67\times10^{-11} \frac{m^3}{kgs^2}$

$g = 9.8 \frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$RadiusEarth = 6.4\times10^6 \; m$

$MassEarth = 5.98\times10^{24} \; kg$

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- One Dimensional Motion

$$\text{displacement} = \Delta x$$

$$\text{average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v(t) = \frac{dx(t)}{dt} \text{ (instantaneous)}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2 x(t)}{dt^2} \text{ (instantaneous)}$$

- Motion with constant a (one dimensional)

$$x(t) = x_o + v_{xo} t + \frac{1}{2} a t^2$$

$$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$$

$$v^2 = v_{xo}^2 + 2a(x - x_o)$$

- Projectile Motion

For motion over level ground :

$$Range = \frac{v_o^2 \sin(2\theta_o)}{g}$$

- X-Y Plane Motion (with constant acceleration)

$$x(t) = x_o + v_{xo} t + \tfrac{1}{2} a_x t^2$$

$$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$$

$$y(t) = y_o + v_{yo} t + \tfrac{1}{2} a_y t^2$$

$$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$$

- Circular Motion

$$\text{Arc Length} = R\Delta\theta$$

$$\omega = \frac{d\theta}{dt} \quad , \quad v_{\tan} = \omega R$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad \omega = 2\pi f$$

$$a_{centripetal} = \frac{v^2}{R} = R\omega^2$$

- Dot, Cross Product

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \bullet \vec{B} = (A_x B_x + A_y B_y + A_z B_z) = |A||B| \cos\theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$$= |A||B| \sin\theta$$

- Frictional Forces

$$F_{s-\max} = \mu_s F_N \quad , \quad F_k = \mu_k F_N \quad , \quad \mu_s > \mu_k$$

- Work and Energy

$$\text{Work Energy Theorem} \rightarrow \text{W}_{net} = \Delta KE$$

$$KE = \frac{1}{2} m v^2 \quad , \quad \Delta KE = KE_2 - KE_1$$

$$\text{One Dimension, F constant :W} = \text{F}\Delta\text{x}$$

$$\text{More Generally : W} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{\text{F}} \bullet \text{d}\vec{r}$$

$$\text{Power : } P = \frac{dW}{dt} \text{ (Watts)}$$

$$\text{1-D : } P = Fv$$

- Potential Energy

$$U(x) = U_o(x_o) - \int_{x_o}^{x} F dx \quad , \quad F(x) = -\frac{dU(x)}{dx}$$

$$\text{Gravity of Earth's Surface : U(y)} = \text{U}_\text{o} + \text{mg}\Delta\text{y}$$

General Expression for Gravitational Potential :

$$U(r) = -\frac{GM_1 M_2}{r}$$

$$\text{For Springs : } U(x) = \frac{1}{2} k x^2$$

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- Conservative Energy Systems

$$E_{tot} = K + U \text{ (constant)}$$

- Center of Mass

$$\vec{r}_{CM} = \frac{1}{M_{total}}\left[m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots m_n\vec{r}_n\right] = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

$$\text{for 1D}: x_{cm} = \frac{1}{M_{total}}\left[m_1x_1 + m_2x_2 + m_3x_3 + \ldots m_nx_n\right]$$

$$\vec{F}_{net} = M_{total}\vec{a}_{CM}$$

- Momentum and Impulse

$$\vec{p} = m\vec{v} \quad , \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{Impulse}: \vec{J} = \int \vec{F}dt = F_{ave}\Delta t = \Delta\vec{p}$$

If no external Forces (only internal Forces) then momentum is conserved.

- 1-D Elastic Collisions (2masses)

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \text{ KE, } \vec{\text{p}} \text{ conserved}$$

- Rotational Motion (const. α)

$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega_o + \alpha t$$

$$\alpha(t) = \alpha \text{ (constant)}$$

- Rotational Motion (general)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad KE_{rot} = \frac{1}{2}I\omega^2$$

$$I = \sum_i m_i R_i^2 = \int \vec{r}^{\,2}$$

$$\text{Parallel Axis Theorem}: I_{\parallel} = I_{CM} + Mh^2$$

$$\vec{\tau} = RF\sin\theta = \vec{R}\times\vec{F} = I\vec{\alpha}$$

$$\vec{L} = I\vec{\omega} = \vec{R}\times\vec{p}$$

$$v_{\tan} = R\omega \quad , \quad a_{\tan} = R\alpha$$

- Kepler's 3rd Law, Force Grav.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \quad , \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$F_{gravity} = \frac{-GM_1M_2}{r^2}$$

- Harmonic Motion

$$F = -kx \quad , \quad U(x) = \frac{1}{2}kx^2 \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos(\omega t + \phi)$$

Damped Harmonic Motion (damp. const. b)

$$x(t) = Ae^{-\gamma t}\cos(\omega' t + \phi)$$

$$\gamma = b/2m \quad , \quad \omega' = \sqrt{k/m - b^2/4m^2}$$

Forced Harmonic Motion (Forcing freq. ω)

$$x(t) = A(\omega)\cos(\omega t + \phi_o) \quad , \quad \omega_o = \sqrt{k/m}$$

$$A(\omega) = \frac{F_o}{m\sqrt{(\omega^2 - \omega_o^2)^2 + b^2\omega^2/m^2}}$$

- Pendulums $\omega = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T}$
- Waves

λ = wavelength, distance between crests (fixed t)

T = Period, time between crests (fixed x)

Equation for wave travelling in + x direction:

$$D(x,t) = A\sin(kx - \omega t) \text{ or } A\cos(kx - \omega t)$$

Equation for wave travelling in – x direction:

$$D(x,t) = A\sin(kx + \omega t) \text{ or } A\cos(kx + \omega t)$$

$$\lambda = \frac{2\pi}{k}$$

$$v_{wave} = \lambda f \quad , \quad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\text{For Strings with tension T}: v = \sqrt{\frac{T}{\mu}}$$