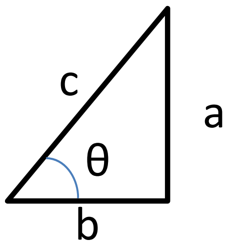
95.141 Fall 2009 Final: Formula Sheet

- Trig



$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

$$c^2 = a^2 + b^2$$

- Quadratic Formula

$ax^2 + bx + c = 0$ has solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Misc. Formulas

Circumference of a circle: $2\pi r$

Area of a Circle: πr^2

Surface Area of a Sphere: $4\pi r^2$

Volume of a Sphere: $\frac{4}{3}\pi r^3$

Volume of a Cylinder: $h\pi r^2$

- Derivatives

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}\ (n \neq 0)$$

$$\frac{d}{dx}\left(\cos ax\right) = -a\sin ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\sin ax\right) = a\cos ax \text{ (ax in radians)}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x} \quad , \quad \frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\text{Chain Rule}: \frac{d}{dx}\left(f(g(x))\right) = \frac{df(g)}{dg}\frac{dg(x)}{dx}$$

$$\text{Distributive Rule}: \frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

- Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + C$$

$$\int \frac{dx}{x} = \ln x + C$$

$$\int \sin x\, dx = -\cos x + C$$

$$\int \cos x\, dx = \sin x + C$$

- Conversions

$1\text{ liter} = 1\times10^{-3} m^3$, $1\text{ ft} = 0.3048m$

$1\text{ mile} = 1609m$, $1\text{ mile per hour} = 0.447\ m/s$

$1mm = 0.001m$, $1g = 0.001kg$

- Units

Velocity $\rightarrow dx/dt = m/s$

Acceleration $\rightarrow d^2x/dt^2 = m/s^2$

Force $\rightarrow$ Newton (N) $= kgm/s^2$

Momentum $\rightarrow kgm/s$

Torque $\rightarrow$ N-m

Angular Momentum $\rightarrow kgm^2/s$

SI Units [mass, length, time] = [kg, m, s]

- Constants

$G = 6.67\times10^{-11}\ \frac{m^3}{kgs^2}$

$g = 9.8\ \frac{m}{s^2}$

$\pi \approx 3.1416$

$e \approx 2.718$

$RadiusEarth = 6.4\times10^6\ m$

$MassEarth = 5.98\times10^{24}\ kg$

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- One Dimensional Motion

$$\text{displacement} = \Delta x$$

$$\text{average velocity} = \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$\text{average acceleration} = \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

$$v(t) = \frac{dx(t)}{dt} \text{ (instantaneous)}$$

$$a(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ (instantaneous)}$$

- Motion with constant a (one dimensional)

$$x(t) = x_o + v_{xo}t + \frac{1}{2}at^2$$

$$v(t) = v_{xo} + at \quad , \quad a(t) = \text{constant}$$

$$v^2 = v_{xo}^2 + 2a(x - x_o)$$

- Projectile Motion

For motion over level ground:

$$Range = \frac{v_o^2 \sin(2\theta_o)}{g}$$

- X-Y Plane Motion (with constant acceleration)

$$x(t) = x_o + v_{xo}t + \tfrac{1}{2}a_x t^2$$

$$v_x(t) = v_{xo} + a_x t \quad , \quad a_x(t) = a_x \text{ (constant)}$$

$$y(t) = y_o + v_{yo}t + \tfrac{1}{2}a_y t^2$$

$$v_y(t) = v_{yo} + a_y t \quad , \quad a_y(t) = a_y \text{ (constant)}$$

- Circular Motion

$$\text{Arc Length} = R\Delta\theta$$

$$\omega = \frac{d\theta}{dt} \quad , \quad v_{\tan} = \omega R$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad \omega = 2\pi f$$

$$a_{centripetal} = \frac{v^2}{R} = R\omega^2$$

- Dot, Cross Product

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \bullet \vec{B} = (A_xB_x + A_yB_y + A_zB_z) = |A||B|\cos\theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$= |A||B|\sin\theta$$

- Frictional Forces

$$F_{s-\max} = \mu_s F_N \quad , \quad F_k = \mu_k F_N \quad , \quad \mu_s > \mu_k$$

- Work and Energy

$$\text{Work Energy Theorem} \rightarrow \text{W}_{net} = \Delta KE$$

$$KE = \frac{1}{2}mv^2 \quad , \quad \Delta KE = KE_2 - KE_1$$

$$\text{One Dimension, F constant :W} = \text{F}\Delta\text{x}$$

$$\text{More Generally: W} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{\text{F}} \bullet \text{d}\vec{r}$$

$$\text{Power}: P = \frac{dW}{dt} \text{ (Watts)}$$

$$\text{1-D}: P = Fv$$

- Potential Energy

$$U(x) = U_o(x_o) - \int_{x_o}^{x} F dx \quad , \quad F(x) = -\frac{dU(x)}{dx}$$

$$\text{Gravity of Earth's Surface: U(y)} = \text{U}_\text{o} + \text{mg}\Delta\text{y}$$

General Expression for Gravitational Potential:

$$U(r) = -\frac{GM_1M_2}{r}$$

$$\text{For Springs}: U(x) = \frac{1}{2}kx^2$$

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- Conservative Energy Systems

$$E_{tot} = K + U \text{ (constant)}$$

- Center of Mass

$$\vec{r}_{CM} = \frac{1}{M_{total}}\left[m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots m_n\vec{r}_n\right] = \frac{\sum_i \vec{r}_i m_i}{\sum_i m_i}$$

$$\text{for 1D}: x_{cm} = \frac{1}{M_{total}}\left[m_1x_1 + m_2x_2 + m_3x_3 + \ldots m_nx_n\right]$$

$$\vec{F}_{net} = M_{total}\vec{a}_{CM}$$

- Momentum and Impulse

$$\vec{p} = m\vec{v} \quad , \quad \vec{F} = \frac{d\vec{p}}{dt}$$

$$\text{Impulse}: \vec{J} = \int \vec{F}dt = F_{ave}\Delta t = \Delta\vec{p}$$

If no external Forces (only internal Forces) then momentum is conserved.

- 1-D Elastic Collisions (2masses)

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \text{ KE}, \vec{\text{p}} \text{ conserved}$$

- Rotational Motion (const. α)

$$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega(t) = \omega_o + \alpha t$$

$$\alpha(t) = \alpha \text{ (constant)}$$

- Rotational Motion (general)

$$T = \frac{1}{f} = \frac{2\pi}{\omega} \quad , \quad KE_{rot} = \frac{1}{2}I\omega^2$$

$$I = \sum_i m_i R_i^2 = \int \vec{r}^{\,2}$$

$$\text{Parallel Axis Theorem}: I_{\parallel} = I_{CM} + Mh^2$$

$$\vec{\tau} = RF\sin\theta = \vec{R}\times\vec{F} = I\vec{\alpha}$$

$$\vec{L} = I\vec{\omega} = \vec{R}\times\vec{p}$$

$$v_{\tan} = R\omega \quad , \quad a_{\tan} = R\alpha$$

- Kepler's 3rd Law, Force Grav.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \quad , \quad \frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

$$F_{gravity} = \frac{-GM_1M_2}{r^2}$$

- Harmonic Motion

$$F = -kx \quad , \quad U(x) = \frac{1}{2}kx^2 \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

$$x(t) = A\cos(\omega t + \phi)$$

Damped Harmonic Motion (damp. const. b)

$$x(t) = Ae^{-\gamma t}\cos(\omega' t + \phi)$$

$$\gamma = b/2m \quad , \quad \omega' = \sqrt{k/m - b^2/4m^2}$$

Forced Harmonic Motion (Forcing freq. ω)

$$x(t) = A(\omega)\cos(\omega t + \phi_o) \quad , \quad \omega_o = \sqrt{k/m}$$

$$A(\omega) = \frac{F_o}{m\sqrt{(\omega^2 - \omega_o^2)^2 + b^2\omega^2/m^2}}$$

- Pendulums $\omega = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{T}$
- Waves

λ = wavelength, distance between crests (fixed t)

T = Period, time between crests (fixed x)

Equation for wave travelling in + x direction:

$$D(x,t) = A\sin(kx - \omega t) \text{ or } A\cos(kx - \omega t)$$

Equation for wave travelling in − x direction:

$$D(x,t) = A\sin(kx + \omega t) \text{ or } A\cos(kx + \omega t)$$

$$\lambda = \frac{2\pi}{k}$$

$$v_{wave} = \lambda f \quad , \quad T = \frac{1}{f} = \frac{2\pi}{\omega}$$

$$\text{For Strings with tension T}: v = \sqrt{\frac{T}{\mu}}$$