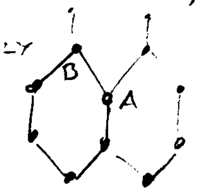
HW #1 Solutions

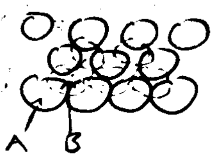
1)

a)

NOT BRAVAIS LATTICE, LOOKS COMPLETELY DIFFERENT FROM PTS. A + B



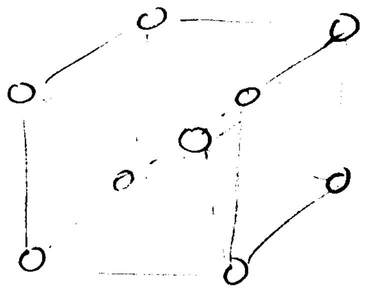
b) 1st LAYER



2nd LAYER

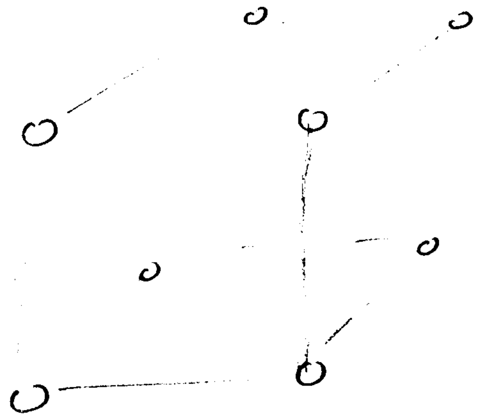
NOT BRAVAIS → BALL "A" SEES BALL "B", BUT BALL "B" HAS NO EQUIVALENT BALL THAT IT SEES.

c)



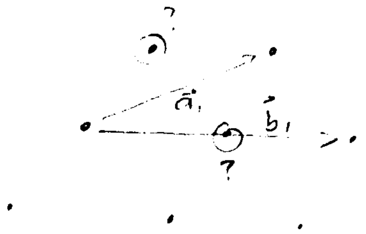
YES → BRAVAIS LATTICE!

d) SIMPLE CUBIC



2)

$\vec{a}_1, \vec{b}_1 \Rightarrow$

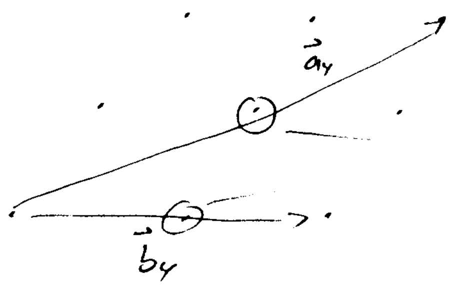


HOW CAN YOU ACCESS CIRCLED POINTS W/ $\vec{a}_1, \vec{b}_1$?? YOU CAN'T, SO $\vec{a}_1, \vec{b}_1$ NOT P.V.'S

$\vec{a}_2, \vec{b}_2$ YES, PV's!

$\vec{a}_3, \vec{b}_3$ NO, CAN ONLY ACCESS PT'S ALONG $\vec{a}_3, \vec{b}_3 \rightarrow$ VECTORS ARE COLINEAR, CAN'T SPAN 2D LATTICE!

$\vec{a}_4, \vec{b}_4$ <u>NO</u>



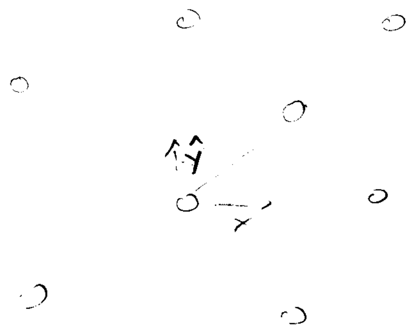
HOW CAN YOU ACCESS THESE LP'S?

4) Coordination #'s for

1) FCC → closest LP's to origin are at

$$\left.\begin{array}{l} \frac{a}{2}\hat{x} + \frac{a}{2}\hat{y} \\ \frac{a}{2}\hat{x} - \frac{a}{2}\hat{y} \\ -\frac{a}{2}\hat{x} + \frac{a}{2}\hat{y} \end{array}\right\} \text{XY plane}$$

$$-\frac{a}{2}\hat{x} - \frac{a}{2}\hat{y}$$



Same # for YZ, ZX planes

$\boxed{\text{Coord. \# (FCC)} = 12}$

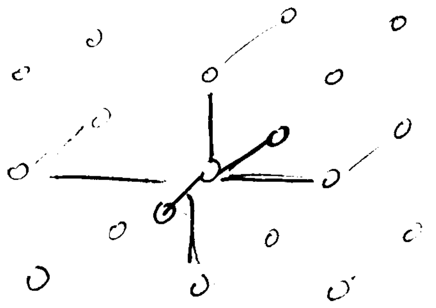
2) BCC | Closest LP is body-centered pt.

There are 4 of these, so

coord. # = 4

3) Simple cubic ⇒ closest pts are nearest corners

Coord # = 6



5) PV's for BCC if $a = 5.65$ Å

Can use various Prim. Vectors

① $a_1 = a\hat{x}$
$a_2 = a\hat{y}$
$a_3 = a/2\,(\hat{x}+\hat{y}+\hat{z})$

or

② $a_1 = \frac{a}{2}(\hat{y}+\hat{z}-\hat{x})$
$a_2 = \frac{a}{2}(\hat{z}+\hat{x}-\hat{y})$
$a_3 = \frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$

Use ② (Either will work)

$$\vec{b}_1 = 2\pi \frac{\vec{a}_2 \times \vec{a}_3}{\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3)}$$

$$\vec{a}_2 \times \vec{a}_3 = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \cdot \frac{a^2}{4} (0\hat{x} + 2\hat{y} + 2\hat{z}) = \frac{a^2}{2}(\hat{y}+\hat{z})$$

$$\vec{a}_1 \cdot (\vec{a}_2 \times \vec{a}_3) = \frac{a^3}{4}(1+1) = \boxed{\frac{a^3}{2}}$$

$$\vec{a}_3 \times \vec{a}_1 = \frac{a^2}{4}\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = \frac{a^2}{4}(2\hat{x} + 0\hat{y} + 2\hat{z}) = \frac{a^2}{2}(\hat{x}+\hat{z})$$

$$\vec{a}_1 \times \vec{a}_2 = \frac{a^2}{4}\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = \frac{a^2}{4}(2\hat{x} + 2\hat{y} + 0\hat{z}) = \frac{a^2}{2}(\hat{x}+\hat{y})$$

$$\vec{b}_1 = \frac{2\pi \frac{a^2}{2}(\hat{y}+\hat{z})}{a^3/\cancel{2}} = \boxed{\frac{2\pi}{a}(\hat{y}+\hat{z})}$$

$$\vec{b}_2 = \frac{2\pi}{a}(\hat{x}+\hat{z}) \qquad \vec{b}_3 = \frac{2\pi}{a}(\hat{x}+\hat{y})$$

THESE ARE PV's FOR FCC LATTICE!

6) Simple cubic lattice $\vec{a}_1 = a\hat{x}$ $\vec{a}_2 = a\hat{y}$ $\vec{a}_3 = a\hat{z}$

Basis $\Rightarrow$ $\vec{0}$, $\frac{a}{2}(\hat{x}+\hat{y})$, $\frac{a}{2}(\hat{y}+\hat{z})$, $\frac{a}{2}(\hat{z}+\hat{x})$

7) a) $3\hat{x} + 4\hat{y} + 2\hat{z}$

$\Downarrow$

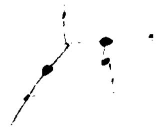
$\frac{1}{3}, \frac{1}{4}, \frac{1}{2}$

$\Downarrow$

$4, 3, 6$ $\boxed{(436)}$

b) (100)

c)



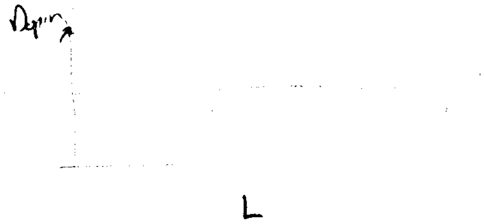
Intercepts of z axis at

$(0,0,-1)$

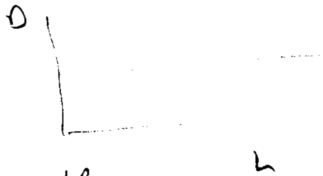
$\boxed{(11\bar{1})}$

8) $K_d = \frac{C_L}{C_S} = 0.3$

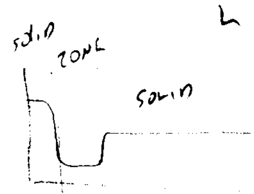
Initial doping profile



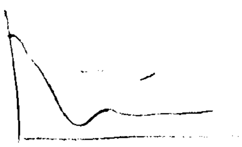
Melt front portion of ingot



Start moving zone



Continue



9]

CZOCHRALSKI

PRO'S

CHEAP
FAST
HIGH QUALITY
UNIFORM
LARGE AREA/VOLUME

CONS

NOT HIGH ENOUGH PURITY?
UNIFORM → CAN'T HAVE 2 TYPES OF SEMICONDUCTOR
NO SPATIAL RESOLUTION

MBE

PROS

VERY ACCURATE
MANY TYPES OF X-TAL
VERY HIGH QUALITY

CONS

$ $ $
SLOW
CAN'T DO IT W/OUT STARTING WAFER

10)

$$f_n = Ce^{-E_n/KT}$$

$$\sum_n Ce^{-E_n/KT} = 1$$

$$x = -hv/KT$$

$$\sum_n C(e^{x})^n = 1$$

This is a geometric sum of the form

$$\sum_n^{\infty} y^n \quad \text{where } y<1$$

so $$\sum_n^{\infty} y^n = \frac{1}{1-y}$$

$$C\,\frac{1}{1-e^{x}} = 1$$

$$\underline{C = 1-e^{-hv/KT}}$$

$$\bar{E} = \sum_n^{\infty} E_n \cdot f_n(E) = \sum_n^{\infty} Cnhv\,e^{-E_n/KT} = \sum_n^{\infty} Cnhv\,e^{-nhv/KT}$$

$$x = hv/KT$$

$$\bar{E} = Chv \sum_n^{\infty} n e^{-xn}$$

$$\frac{\partial}{\partial x}\sum -e^{-xn}$$

$$\frac{\partial}{\partial x}\left[\frac{-1}{1-e^{x}}\right] \cdot \frac{1 \cdot e^{-x}}{(1-e^{-x})^2}$$

10| CONT

$$\bar{E} = \frac{C h\nu e^{-x}}{(1-e^{-x})^2} = \frac{\overbrace{\cancel{(1-e^{-h\nu/kT})}}^{C} h\nu e^{-h\nu/kT}}{(1-e^{-h\nu/kT})^{2}}$$

$$\bar{E} = \frac{h\nu e^{-h\nu/kT}}{1-e^{-h\nu/kT}} = \frac{h\nu}{e^{h\nu/kT}-1}$$

11|

a) $\Phi_F = 4.7eV$

$\lambda = 632nm \Rightarrow E = 1.96eV$

$h\nu = \Phi_F + KE_{e^-}$

$1.96eV = 4.7eV + KE_{e^-}$

NEGATIVE KE => NOT POSSIBLE, NO e^- EJECTED!

b) $\nu = 2\times10^{15}$ Hz => 8.3eV

$8.3eV = 4.7eV + KE_{e^-}$

$$\boxed{KE_{e^-} = 3.6eV}$$

12)

$L = n\hbar$

$F = ma$

$$\frac{-e^2}{4\pi\epsilon_0 r^2} = -\frac{mv^2}{r}$$

$L = n\hbar = mvr$

$mv = \frac{n\hbar}{r}$

$$mv^2 = \frac{n^2\hbar^2}{mr^2}$$

$$\frac{-e^2}{4\pi\epsilon_0 r^2} = \frac{-n^2\hbar^2}{mr^3}$$

$$r_n = \frac{n^2\hbar^2 4\pi\epsilon_0}{me^2} \Rightarrow \boxed{r_n = 0.53\text{Å} \cdot n^2}$$

$n\hbar = mvr \qquad mv = \frac{h}{\lambda}$

$$n\hbar = \frac{hr}{\lambda}$$

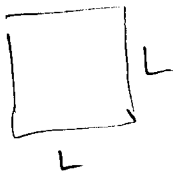
$$\frac{nh}{2\pi} = \frac{hr}{\lambda}$$

$$\lambda = \frac{2\pi r_n}{n}$$

$$\lambda_n = \frac{2\pi}{n} \cdot \frac{n^2\hbar^2 4\pi\epsilon_0}{me^2}$$

$$\boxed{\lambda_n = \frac{2\pi n \hbar^2 4\pi\epsilon_0}{me^2}}$$

13)



SOLUTION TO W.E

$$n_x^2 + n_y^2 = \frac{4L^2}{\lambda^2}$$

$$N = \underbrace{\tfrac{1}{4}}_{\text{ONLY POSITIVE}} \cdot \underbrace{2}_{\text{polarization}} \cdot \pi\left(\sqrt{n_x^2+n_y^2}\right)^2 = \frac{\pi}{2}\left(n_x^2+n_y^2\right)$$

$$N = \frac{\pi}{2}\,\frac{4L^2}{\lambda^2} = \frac{2\pi L^2}{\lambda^2}$$

$$n = N/V = \frac{2\pi}{\lambda^2}$$

$$\boxed{\partial n/\partial \lambda = \frac{4\pi}{\lambda^3}}$$

14)

$$\psi(x) = A\sin\frac{2\pi}{L}x \qquad 0 < x < L$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi = E\psi$$

$$+\frac{\hbar^2}{2m}\cdots\frac{4\pi^2}{L^2}\cancel{A\sin\frac{2\pi}{L}x} = E\cancel{A\sin\frac{2\pi}{L}x}$$

$$\boxed{E = \frac{4\pi^2\hbar^2}{2m}}$$ ✓

$$A = \sqrt{\frac{2}{L}}$$

15

$$\langle x \rangle = \int_0^L \sqrt{\frac{2}{L}}^2 \sin\frac{2\pi x}{L}\; x \; \sin\frac{2\pi x}{L}\, dx$$

$$= \frac{x^2}{4} - \frac{x \sin \frac{4\pi x}{L}}{8\pi/L} - \frac{\cos \frac{4\pi}{L}x}{8\cdot\left(\frac{2\pi}{L}\right)^2}\Bigg|_0^L$$

$$= \left[\frac{L^2}{4} - 0 - \frac{\cancel{L^2}}{\cancel{32}\pi^2}\right] - \left[0 - 0 - \frac{\cancel{L^2}}{\cancel{32}\pi^2}\right]$$

$$= \frac{2}{L}\cdot\frac{L^2}{4} = \boxed{\frac{L}{2}}$$

$$\boxed{\langle x \rangle^2 = \frac{L^2}{4}}$$

$$\langle x^2 \rangle = \int_0^L \frac{2}{L} \sin\frac{2\pi x}{L}\; x^2 \; \sin\frac{2\pi x}{L}\, dx$$

$$\frac{1}{a^3}\left[a^2 \cdot x^2 \left(-\frac{\cos ax \sin ax}{2} + \frac{ax}{2}\right) - \frac{ax\cos(ax)}{2} + \frac{\cancel{\cos ax \sin ax}}{4} + \frac{ax}{4} - \frac{1}{3}a^3x^3\right]$$

$$\frac{1}{a^3}\left[\frac{a^3x^3}{2} - \frac{ax}{4} - \frac{a^3x^3}{3}\right]_0^L \cdot \frac{2}{L}$$

$$= \frac{L^3}{8\pi^3}\left[\frac{8\pi^3 \cdot \cancel{L^3}}{2\cdot \cancel{L^3}} - \frac{2\pi \cancel{L}}{4\cancel{L}} - \frac{8\pi^3\,\cancel{L^3}}{3\cancel{L^3}}\right]\cdot\frac{2}{L}$$

$$= L^2\left[1 - \frac{1}{8\pi^2} - \frac{2}{3}\right] \cong \boxed{L^2/3}$$

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$
$$= 0.16\,L$$

$\Delta x = 0.4 L$