HW #4

1) For GaAs $\hbar\omega_{LO} = 36\,meV$ near $K=0$

For a linear diatomic chain

$$\omega_q^2 = \frac{K}{\mu}\left[1 \pm \sqrt{1 - \frac{4\mu^2}{mM}\sin^2\frac{qa}{2}}\right]$$

For upper branch, near $q=0$

$$\omega^2(q \approx 0) = \frac{K}{\mu}(2) \qquad \mu = \left(\frac{1}{m} + \frac{1}{M}\right)^{-1}$$

Atomic Mass Ga = 69.723 amu ⇒ 1.16×10^{-25} kg

Atomic Mass As = 74.92 amu ⇒ 1.24×10^{-25} kg

$$\mu = 5.99 \times 10^{-26}\ kg$$

$$36\,meV \cdot 1.6 \times 10^{-19} = \hbar\sqrt{\frac{2K}{\mu}}$$

$$K = \left[\frac{.036 \times 1.6 \times 10^{-19}}{\hbar}\right]^2 \cdot \frac{\mu}{2}$$

$$\boxed{K = 90\ N/m}$$

b)

In $\rightarrow$ 114.818 amu $\Rightarrow$ 1.91×10^{-25} Kg

P $\rightarrow$ 30.974 amu $\Rightarrow$ 5.14×10^{-26} Kg

$$\omega_q(InAs) = \sqrt{\frac{2K}{\mu_{InAs}}}$$

$$\mu_{InAs} = \left(\frac{1}{1.24\times10^{-25}} + \frac{1}{1.91\times10^{-25}}\right)^{-1}$$

$$\omega_q(InAs) = \sqrt{\frac{2\cdot 90.1}{7.52\times10^{-26}}}$$

$$\mu_{InAs} = 7.52\times10^{-26} \text{ Kg}$$

$$\omega_q(InAs) = 4.893\times10^{13}$$

$$\nu_q(InAs) = 7.79\times10^{12} \text{ Hz}$$

$$E_q(InAs) = 32.1 \text{ meV}$$

$$\mu_{InP} = 4.05\times10^{-26} \text{ Kg}$$

$$\mu_{GaP} = 3.56\times10^{-26} \text{ Kg}$$

$$\nu_q(InP) = 1.05\times10^{13} \text{ Hz}$$

$$E(InP) = 43 \text{ meV}$$

$$\nu_q(GaP) = 1.125\times10^{13} \text{ Hz}$$

$$E(GaP) = 46 \text{ meV}$$

c)

From www.ioffe.ru/SVA/NSM/Semicond/GaP/mechanic.html

InP $\nu_{LO} = 1.03\times10^{12}$ Hz

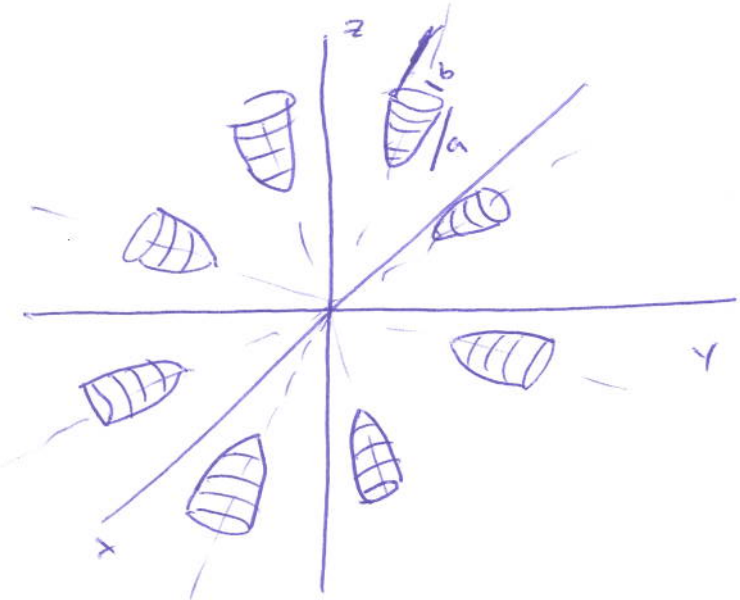
InAs $\nu_{LO} = 7.01\times10^{12}$ Hz

GaP $\nu_{LO} = 11.3\times10^{12}$ Hz

Almost an exact match!!

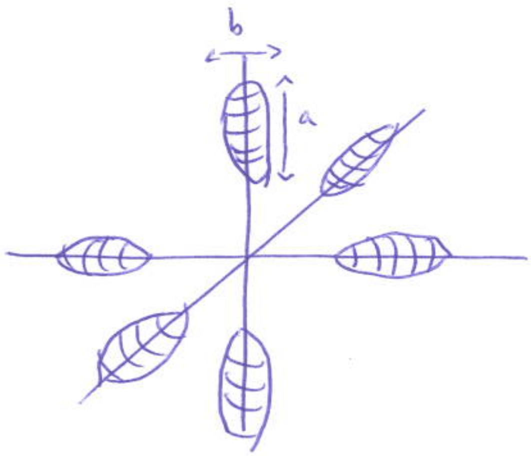
(2)

Ge



$$\frac{a}{b} = \sqrt{\frac{m_l^*}{m_t^*}}$$

Si

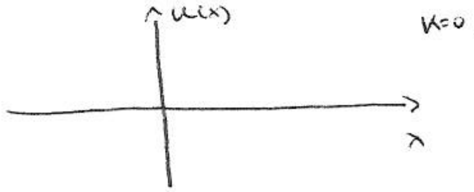


$$\frac{a}{b} = \sqrt{\frac{m_l^*}{m_t^*}}$$

3]

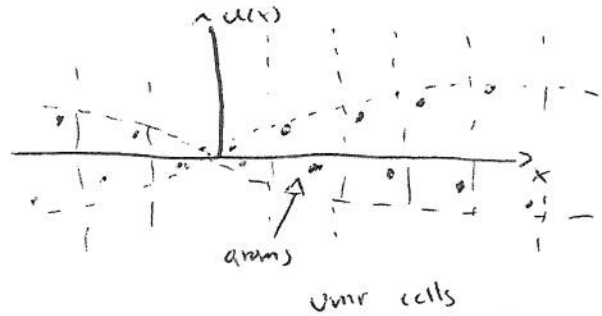
$k=0$

Acoustic

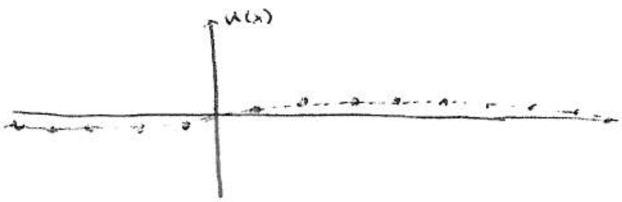


OPTICAL

$K = 0 \qquad \lambda \sim \infty$



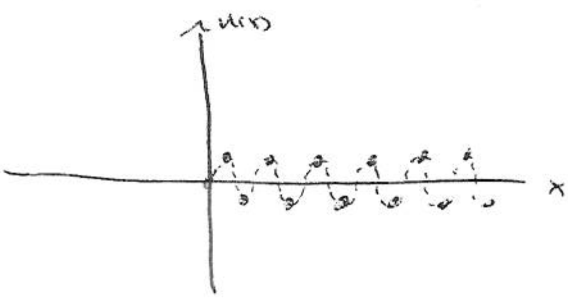
$k \approx 0$



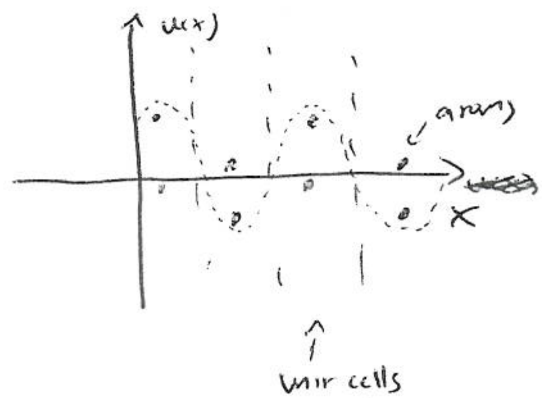
VERY LONG λ

$k = \pi/a \qquad \lambda = 2a$

Acoustic



OPTICAL



3] $k=0$, Acoustic + Optical @

4) What is U_0?

$$V = 1mm \times 1mm \times 1mm = (10^{-3})^3 m^{-3} = 10^{-9} m^{-3}$$

For N atoms

$$E_{total} = N \cdot \tfrac{1}{2} m U_0^2 \omega_q^2$$

$$\omega_q = \left(\frac{36meV}{1000}\right)\left(1.6\times10^{-19}\, J/eV\right) / \hbar = 5.5\times10^{13}\, rad/s$$

atoms in $1cm^{+3}$ GaAs $= 4.42\times10^{22}$

in $1mm^3$ $= 4.42\times10^{25}$

$$E_{total} = 4.42\times10^{25} \cdot \tfrac{1}{2} \cdot m U_0^2 \omega_q^2 = 36meV$$

$$\frac{2 \cdot 5.76\times10^{-21}\, J}{4.42\times10^{25} \cdot 5.99\times10^{-26} \cdot (5.5\times10^{13})^2} = U_0^2$$

$$U_0 = 1.199\times10^{-24}\, m$$