# MODERN PHYSICS
## HOMEWORK 4 SOLUTION

4-1 $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $R = 1.097 \times 10^7 m^{-1}$

The Lyman series end on $m=1$, the Balmer series on $m=2$, and the Paschen series on $m=3$. The series limit all have $n=\infty$, so $\frac{1}{n}=0$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2}\right) = 1.097 \times 10^7 m^{-1}$$

$$\lambda_L (\text{limit}) = \frac{1}{1.097 \times 10^7 m^{-1}} = 91.16 \times 10^{-9} m$$

$$= 91.16 \text{ nm}$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2}\right) = 1.097 \times 10^7 m^{-1} / 4$$

$$\lambda_B = \frac{4}{1.097 \times 10^7 m^{-1}} = 3.646 \times 10^{-7} m = 364.6 nm$$

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2}\right) = \frac{1.097 \times 10^7 m^{-1}}{9} \rightarrow \lambda_P = \frac{9}{1.097 \times 10^7 m^{-1}} = 8.204 \times 10^{-7} m$$

$$= 820.4 \text{ nm}$$

4-2 $$\frac{1}{\lambda_m} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

where $m=2$ for Balmer series

$$\frac{1}{379.1\,nm} = \frac{1.097 \times 10^7\ m^{-1}}{10^9\ nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^9\ nm/m}{379.1\ nm(1.097 \times 10^7\ m^{-1})} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095}$$

$$n = \left(\frac{1}{0.0095}\right)^{\frac{1}{2}} = 10.3 \quad \rightarrow n = 10$$

$n = 10 \rightarrow n = 2$

4-6 (a) $f = \pi b^2 n t$

For Au, $n = 5.90 \times 10^{28}$ atoms/$m^3$

and for this foil $t = 2.0\,\mu m = 2.0 \times 10^{-6} m$

$$b = \frac{k q_\alpha Q}{m_\alpha v^2} \cot\frac{\theta}{2} = \frac{2(79) ke^2}{2 K\alpha} \cot\frac{90}{2}$$

$$= \frac{2(79)(1.44\ eV.nm)}{2(7.0 \times 10^6\ eV)} = 1.63 \times 10^{-5}\ nm$$

$$= 1.63 \times 10^{-14} m$$

$$f = \pi (1.63 \times 10^{-14} m)^2 (5.90 \times 10^{28}/m^3)(2.0 \times 10^{-6} m)$$

$$= 9.8 \times 10^{-5}$$

(b) For $\theta = 45°$, $b(45°) = b(90°) \frac{\left(\cot\frac{45°}{2}\right)}{\left(\cot\frac{90°}{2}\right)}$

$$= b(90°) \frac{(\tan 90°/2)}{\left(\tan\frac{45°}{2}\right)}$$

$$= 3.92 \times 10^{-5}\ nm = 3.92 \times 10^{-14} m$$

and $f(45°) = 5.7 \times 10^{-4}$

For $\theta = 75°$

$$b(75°) = b(90°) \frac{\left(\tan \frac{90°}{2}\right)}{\left(\tan\left(\frac{75°}{2}\right)\right)}$$

$$= \cancel{3.92 \times 10^{-5}\,nm}$$

$$= 2.12 \times 10^{-5}\,nm = 2.12 \times 10^{-14}\,m$$

and $f(75°) = 1.66 \times 10^{-4}$

Therefore, $\Delta f(45° - 75°) = 5.7 \times 10^{-4} - 1.66 \times 10^{-4}$

$$= 4.05 \times 10^{-4}$$

(c) Assuming the Au atom to be a sphere of radius

$$\frac{4}{3}\pi r^3 = \frac{M}{N_A \rho} = \frac{197\,g/mole}{(6.02 \times 10^{23}\,atoms/mole)(19.3\,g/cm^3)}$$

$$r = \left[\frac{3}{4\pi} \frac{197\,g/mole}{(6.02 \times 10^{23}\,atoms/mole)(19.3\,g/cm^3)}\right]^{1/3}$$

$$r = 1.62 \times 10^{-3}\,cm = 1.62 \times 10^{-10}\,m = 16.2\,nm$$

4-12 (a) $f = \pi b^2 n t$

For $\theta = 25°$

$$b = \frac{(2)\,79\,ke^2}{2K_\alpha} \cot \frac{25}{2}$$

$$= \frac{(2)(79)\ (1.44\ eV.nm)}{2\ (7.0\ \times 10^6 eV)} \cot\left(\frac{25°}{2}\right)$$

$$= 7.33\ \times 10^{-5}\ nm = 7.33\ \times 10^{-14}\ m$$

$$f = \pi\ (7.33 \times 10^{-14}\ m)^2\ (5.90\ \times 10^{28}/m^3)\ (2.0\ \times 10^{-6} m)$$

$$= 1.992\ \times 10^{-3}$$

Becouse $\Delta N = f \times N = 1000$

$$N = \frac{1000}{1.992\ \times 10^{-3}} = 5.02\ \times 10^5$$

For $\theta = 45°$, $b = \frac{(2)\,(79)\ (1.44\ eV.nm)}{2\,(7.0 \times 10^6 eV)} \cot\left(\frac{45°}{2}\right)$

$$= 3.92\ \times 10^{-14}\ m$$

$$f = \pi (3.92 \times 10^{-14}\,m)^2 (5.90 \times 10^{28}/m^3)(2.0 \times 10^{-6}\,m)$$
$$= 5.70 \times 10^{-4}$$

Because $\Delta N\ (\theta > 45°) = f \times N$
$$= 5.70 \times 10^{-4} (5.02 \times 10^{5}) = 286$$

(b) $\Delta N\ (25° \rightarrow 45°) = 1000 - 286 = 714$

(c) For $\theta = 75°$, $b = b(\theta > 25°) \dfrac{\left(\tan \frac{25°}{2}\right)}{\left(\tan \frac{75°}{2}\right)} = 2.12 \times 10^{-14}\,m$

$$f = 1.992 \times 10^{-3} \frac{(2.12 \times 10^{-14}\,m)^2}{(7.33 \times 10^{-14})^2}$$

$$= 1.992 \times 10^{-3} \left(\frac{2.12}{7.33}\right)^2 = 1.67 \times 10^{-4}$$

For $\theta = 90°$, $b = b(\theta > 25°) \dfrac{\left(\tan \frac{25°}{2}\right)}{\left(\tan \frac{90°}{2}\right)} = 1.63 \times 10^{-14} m$

$$f = 1.992 \times 10^{-3} \frac{(1.63 \times 10^{-14} m)^2}{(7.33 \times 10^{-14})^2}$$

$$= 1.992 \times 10^{-3} \left(\frac{1.63}{7.33}\right)^2 = 9.85 \times 10^{-5}$$

$$\Delta N = f \times N = 9.85 \times 10^{-5} (5.02 \times 10^5) = 49$$

$$\Delta N = (75° \rightarrow 90°) = 84 - 49 = 35$$

4-14

$$a_0 = \frac{\hbar^2}{mke^2}$$

$$= \frac{\hbar\hbar}{mcke^2} = \frac{\hbar c}{mc^2} \times \frac{1}{ke^2/\hbar c}$$

$$= \frac{1}{2\pi} \times \frac{h}{mc} \times \frac{1}{ke^2/\hbar c} = \frac{\lambda_c}{2\pi\alpha}$$

$$E_0 = \frac{mk^2e^4}{2\hbar^2}$$

$$= \frac{mc^2\,(ke^2)^2}{2(\hbar c)^2} = \frac{mc^2}{2} \times \left(\frac{ke^2}{\hbar c}\right) = \frac{1}{2}mc^2\alpha^2$$

$$a_0 = \frac{\lambda_c}{2\pi\alpha} = \frac{0.00243\,nm}{2\pi\left(\frac{1}{137}\right)} = 0.053\ nm$$

$$E_0 = \frac{1}{2}mc^2\alpha^2$$

$$= \frac{5.11 \times 10^5\,eV}{2\,(137)^2} = 13.6\ eV$$

4-18 The number of revolutions N in $10^{-8}$ s is:

$$N = 10^{-8}\,s \,/\, (\text{time}/\text{revolution})$$

$$= \frac{10^{-8}\,s}{(\text{circumference of orbit}/\text{speed})}$$

$$N = \frac{10^{-8}\,s}{(C/v)} = \frac{10^{-8}\,s}{(2\pi r/v)}$$

The radius of the orbit is given by:

$$r = \frac{n^2 a_0}{Z} = \frac{4^2\,(0.0529\,nm)}{3}$$

$$C = 2\pi r$$

$$= 2\pi\left[\frac{4^2\,(0.0529\,nm)}{3}\right] = 1.77\,nm = 1.77\times10^{-9}\,m$$

The electron's speed in the orbit is given by:

$$v^2 = \left(\frac{kZe^2}{mr}\right) = \frac{(8.99\times10^{9}\,Nm^2/C^2)\,(3)\,(1.60\times10^{-19}\,C)^2}{(9.11\times10^{-31}\,kg)\,(1.77\times10^{-9}\,m)}$$

$$v = 6.54\times10^{5}\,m/s$$

Therefore,

$$N = \frac{10^{-8}s}{(C/v)} = 3.70 \times 10^{6} \text{ revolutions}$$

4-22 (a) $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

For Lyman $\alpha$ : $\frac{1}{\lambda} = 1.097373 \times 10^{7} \text{ m}^{-1}\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

$\lambda_L = 121.5023 \text{ nm}$

$$E_L = \frac{hc}{\lambda_L} = \frac{1240 \text{ eV.nm}}{121.5023 \text{ nm}} = 10.2056 \text{ eV}$$

and $p_L = \frac{E_L}{c} = 10.2056 \text{ eV}/c$

Conservation of momentum requires that the recoil momentum of the H atom $p_H = p_L$
and the recoil energy $E_H$ is:

$$E_H = \frac{(p_H)^2}{2m_H} = \frac{(p_H c)^2}{2m_H c^2} = \frac{(10.2056 \text{ eV}/c)^2}{2(1.007825 \text{ u}c^2)(931.50 \times 10^{6} \text{eV}/\text{u}c^2)}$$

$$= 5.55 \times 10^{-8} \text{ eV}$$

(b) $\dfrac{E_H}{(E_L + E_H)} \approx \dfrac{5.5 \times 10^{-8}\ eV}{10.29\ eV} = 5 \times 10^{-9}$

4-27. $\dfrac{1}{\lambda} = R\,(Z-1)^2 \left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$= R\,(Z-1)^2 \left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right)$ for $K\alpha$

$$Z-1 = \left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{(0.0794\,nm)\,(1.097 \times 10^{-2}/nm)\,\left(\frac{3}{4}\right)}\right]^{\frac{1}{2}}$$

$Z = 1 + 39.1 \approx 40$ Zirconium

4-34 (a) The available energy is not sufficient to raise ground state electrons to the $n=5$ level which requires $13.6 - 0.54 = 13.1$ eV.

The shortest wavelength (i.e, highest energy) spectral line that will be emitted is the 3rd line of the Lyman series, the $n=4 \rightarrow n=1$ transition

(b) The emitted lines will be for those transitions that begin on the $n=4$, $n=3$, or $n=2$ levels These are the first three lines of the Lyman series, the first two lines of the Balmer series, and the first line of the Paschen series.

4-40 Those scattered at $\theta = 180°$ obeyed the Rutherford formula. This is a head-on collision where the $\alpha$ comes instantaneously to rest before reversing direction. At ~~that~~ that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$$\frac{1}{2} m_\alpha v^2 = 7.7 \text{ MeV} = \frac{k\,(2e)(79e)}{r}$$

where $r$ = upper limit of the nuclear radius.

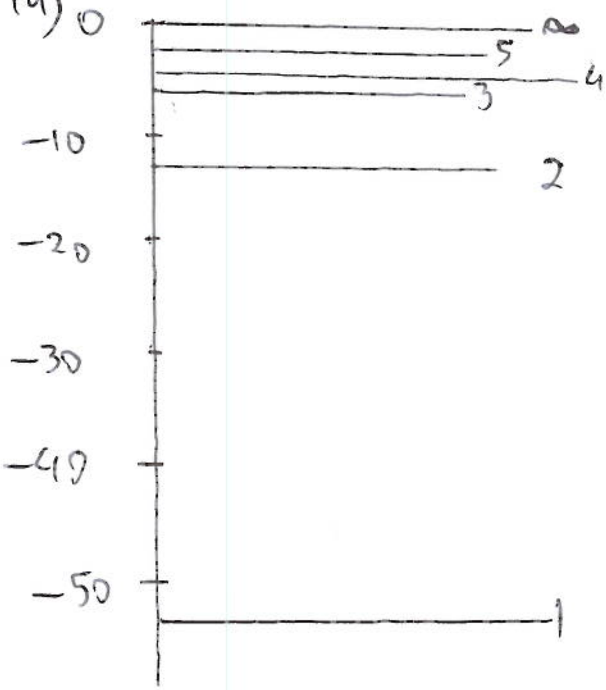
$$r = \frac{k\,(2)(79)\,e^2}{7.7 \text{ MeV}} = \frac{2\,(79)\,(1.440 \text{ MeV}\cdot\text{fm})}{7.7 \text{ MeV}}$$

$$= 29.5 \text{ fm}$$

4-50 For He: $E_n = -\dfrac{13.6\,eV\,Z^2}{n^2}$

$$= -\frac{54.4\,eV}{n^2}$$

(a)




$E_1 = -54.4\,eV$

$E_2 = -13.6\,eV$

$E_3 = -6.04\,eV$

$E_4 = -3.04\,eV$

$E_5 = -2.18\,eV$

$E_\infty = 0\,eV$

(b) Ionization energy is 54.4 eV

(c) H Lyman $\alpha$ : $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\,eV.nm}{(13.6\,eV - 3.4\,eV)} = 121.6\,nm$

H Lyman $\beta$ : $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\,eV.nm}{(13.6\,eV - 1.41\,eV)} = 102.6\,nm$

$He^+$ Balmer $\alpha$ : $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\ eV \cdot nm}{(13.6\ eV - 6.04\ eV)}$

$= 164.0\ nm$

$He^+$ Balmer $\beta$ : $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\ eV \cdot nm}{(13.6\ eV - 3.40\ eV)}$

$= 121.6\ nm$

$\Delta\alpha = 42.4\ nm$

$\Delta\beta = 19.0\ nm$

(The reduced mass correction factor does not change the energies calculated above to three significant figures).

(d) $E_n = -\dfrac{13.6\ eV\ Z^2}{n^2}$, because for $He^+$, $Z = 2$

then $Z^2 = 2^2$. Everytime $n$ is an even number a $2^2$ can be factored out of $n^2$ and cancelled with the $Z^2 = 2^2$ in the numerator; e.g. for $He^+$,

$$E_2 = -\frac{13.6\ eV \cdot 2^2}{2^2} = -13.6\ eV \quad \text{(H ground state)}$$

$$E_4 = -\frac{13.6\ eV \cdot 2^2}{4^2} = -\frac{13.6\ eV}{2^2} \quad \text{(H} - 1^{st} \text{ excited state)}$$

$$E_6 = -\frac{13.6\ eV \cdot 2^2}{6^2} = -\frac{13.6\ eV}{3^2} \quad \text{(H} - 2^{nd} \text{ excited state)}$$

⋮

etc

Thus, all of the H energy level values are to be found within the $He^+$ energy levels, so $He^+$ will have within its spectrum lines that match (nearly) a line in the H spectrum.