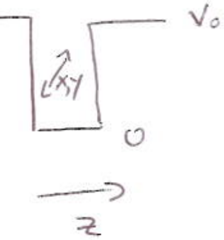
HW #5)

1] 2D DOS



For each z state,

electron energy in z is fixed, free in x, y, except for edges of x-tal

Schrödinger Eq: (IN 2D)

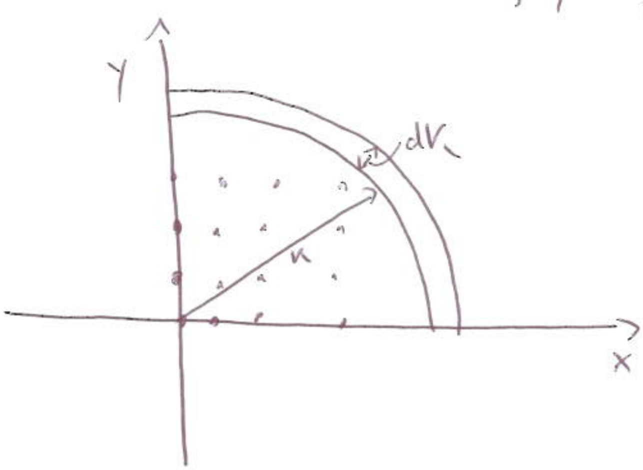
$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + K^2 \psi = 0$$

$$K^2 = \frac{2mE}{\hbar^2} \qquad E_{x,y} = \frac{\hbar^2 k^2}{2m}$$

$$E_{TOTAL} = E_z + E_{x,y} = E_z + \frac{\hbar^2 k_{xy}^2}{2m^*}$$

$$k^2 = k_x^2 + k_y^2 \qquad k_x = \frac{n_x \pi}{a} \qquad k_y = \frac{n_y \pi}{b}$$

$$E_{n_x, n_y} = \frac{\hbar^2}{2m}\left(k_x^2 + k_y^2\right) = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2}\right)$$



$dV_k \Rightarrow dA_k = \frac{\pi}{a} \cdot \frac{\pi}{b}$ volume of k-space for 1 solution

→ ONLY INTERESTED IN 1st QUADRANT

→ ~~ONLY~~ EACH K value has 2 spins

DIFFERENTIAL AREA IN K-SPACE =>

$$dA_k = \underbrace{\tfrac{1}{4}}_{1^{st}\text{ QUAD.}} \; \underbrace{2}_{2\text{ SPINS}} \cdot 2\pi K \cdot dK$$

$$dA_k = \pi K dK$$

ENERGY STATES W/ K BETWEEN $K \to K + dK$ => $\dfrac{dA_k}{\text{Volume/K space}}$

$$= \frac{\pi K dK}{\pi^2/ab} = \frac{K dK\, ab}{\pi}$$

WANT # STATES BETWEEN ~~K → K+dK~~ $E \to E + dE$

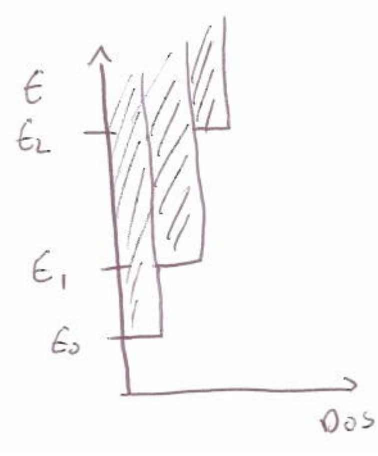
$$E = \frac{\hbar^2 K^2}{2m} \qquad K^2 = \frac{2mE}{\hbar^2}$$

$$dE = \frac{\hbar^2 K dK}{m} \qquad K dK = \frac{dE\, m}{\hbar^2}$$

ENERGY STATES BETWEEN $E \to E + dE$ => $\dfrac{ab}{\pi}\dfrac{m\, dE}{\hbar^2}$

$$\boxed{g(E) = \frac{m\, dE}{\pi \hbar^2}}$$

DENSITY OF STATES IS CONSTANT!



2) DOS m_n^*, m_p^* for Si

$$m_n^* = 6^{2/3} (m_l^* m_t^{*2})^{1/2}$$

$$m_p^* = \left[m_{hh}^{*\,3/2} + m_{lh}^{*\,3/2} \right]^{2/3}$$

CONDUCTION BAND => $m_l = 0.916$

$m_t = 0.1905$

VALENCE BAND => $m_{hh} = 0.537$

$m_{lh} = 0.153$

$$\boxed{m_{n\,Si}^* = 0.602 \cdot m_0}$$

$$\boxed{m_{p\,Si}^* = 0.59\, m_0}$$

3]

$m^* = 0.067$

$E \Rightarrow -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = E\psi$

$$E = \frac{\hbar^2 k^2}{2m} \qquad k = \frac{n\pi}{a}$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2m^* a^2}$$

$$E_n = \frac{\hbar^2 \pi^2}{2m^* a^2}\, n^2$$

$$= 8.9\times10^{-23}\ \mathrm{J} \cdot n^2$$

EACH STATE HOLDS 2 ELECTRONS

a) 13 electrons $n=7$

$$E_7 = E_F = 49 \cdot 8.9\times10^{-23}\ \mathrm{J} = 27\ \mathrm{meV}$$

b) 20 electrons $n=10$ $\boxed{E_F = 55.7\ \mathrm{meV}}$

c) 150 electrons $n=75$ $\boxed{E_F = 3.13\ \mathrm{eV}}$

4)

E_I (P) in Si

$$E_I = \frac{m^* q^4}{2(4\pi \cancel{K_s} \varepsilon_s \varepsilon_0 \hbar)^2} = \frac{13.6}{\varepsilon_s^2} \frac{m^*}{m_0}$$

$$m_n^* \simeq m_n^*(DOS) = 0.6\, m_0$$

$$\frac{m_n^*}{m_0} = 0.6$$

$$\varepsilon_s = 11.8$$

$$\boxed{E_I = 0.058\ eV}$$

$$f_B = A e^{-E/KT}$$

$$\int_0^\infty A e^{-E/KT}\, dE = 1$$

$$-KTAe^{-E/KT}\Big]_0^\infty = 1$$

$$AKT = 1 \qquad A = 1/KT$$

$$f_B = \frac{e^{-E/KT}}{KT} \qquad 0.5 = \frac{e^{-E/KT}}{KT}$$

$$\ln KT/2 = -E/KT$$

$$E = -KT \ln KT/2$$

$$\boxed{T \approx 13K}$$

5]

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$T = 300K$

$k = 1.38\times10^{-23}\ J/K$

$m_n^* = 0.067\, m_0$

$m_p^* = \left[(.51)^{3/2} + (.082)^{3/2}\right]^{2/3}$

$\boxed{m_p^* = 0.53\, m_0}$

$N_v = 9.66\times10^{24}\ m^{-3} = 9.66\times10^{18}\ cm^{-3}$

$N_c = 4.322\times10^{23}\ m^{-3} = 4.3\times10^{17}\ cm^{-3}$

a) $n_0 = N_c\, e^{(E_F - E_c)/kT} = 7.1\times10^{6}\ m^{-3} = 7.1\ cm^{-3}$

$p_0 = N_v\, e^{-(E_F - E_v)/kT} = 8.6\times10^{17}\ m^{-3} = 8.6\times10^{11}\ cm^{-3}$

b) $E_c - E_F = 0.7$

$n_0 = 7.7\times10^{5}\ cm^{-3}$

$p_0 = 7.9\times10^{6}\ cm^{-3}$

c)

$n_0 = 1.9\times10^{14}\ cm^{-3}$

$p_0 = 3.2\times10^{-2}\ cm^{-3}$

6)

$p_0 = 10^{17}\ cm^{-3}$

$p_0 = N_V\, e^{-(E_F - E_V)/KT}$

$N_V = 2\left(\frac{2\pi m_p^* KT}{h^2}\right)^{3/2}$ $\quad m_p^* = 0.59\, m_0$

$N_V = 1.129 \times 10^{19}\ cm^{-3}$

$1 \times 10^{17} = 1.129 \times 10^{19}\, e^{-(E_F - E_V)/KT}$

$-4{,}726 = -(E_F - E_V)/KT$

$$\boxed{E_F - E_V = 0{,}122\,eV}$$