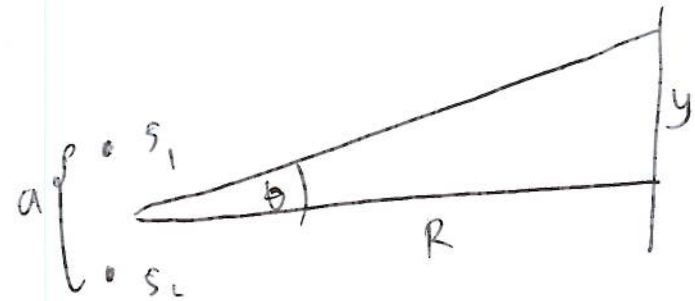
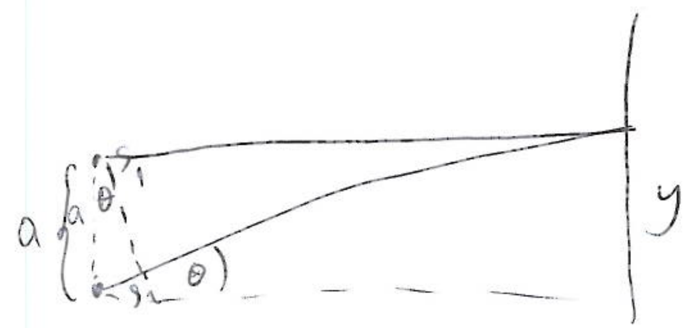
HOMEWORK 5

2. (a) $y_1 = 2\sin(kr - \omega t)$

$y_2 = 2\sin(kr - \omega t)$





for $R > y$

$\sin\theta \approx \frac{y}{R}$

path difference $= a \sin\theta$

$= a \frac{y}{R}$

maxima:

$a \frac{y}{R} = n\lambda$

$y = \frac{n\lambda R}{a}$

minima:

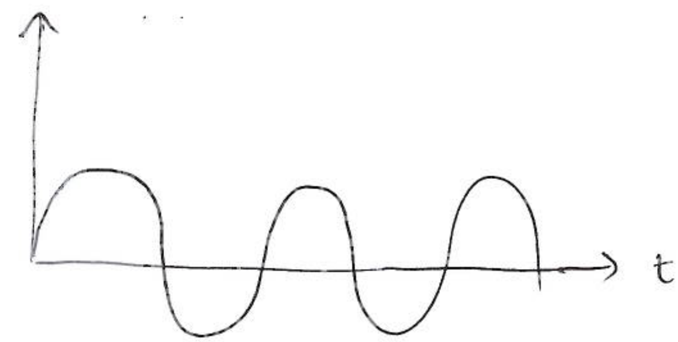
$a \frac{y}{R} = (n + \frac{1}{2})\lambda$

$y = \frac{(n+\frac{1}{2})\lambda R}{a}$

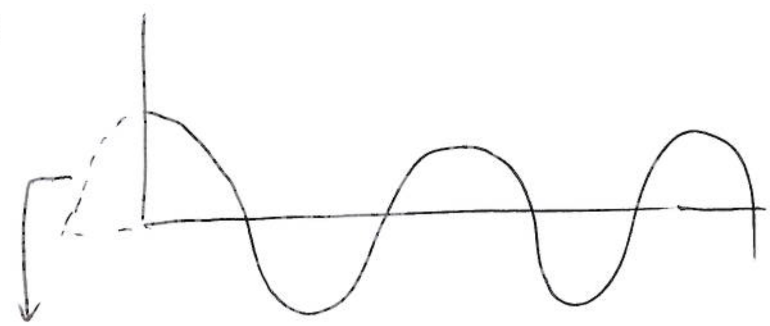
(b) $y_1 = 2\sin(kr - \omega t)$

$y_2 = 2\cos(kr - \omega t)$

$\sin(kr - \omega t)$:



$\cos(kr - \omega t)$:



there is $\frac{1}{4}$ phase difference

path difference $= a \sin\theta + \frac{\lambda}{4}$

maxima: $a \sin\theta + \frac{\lambda}{4} = n\lambda$

$$a \frac{y}{R} + \frac{\lambda}{4} = n\lambda$$

$$a \frac{y}{R} = \left(n - \frac{1}{4}\right)\lambda$$

$$y = \frac{\left(n - \frac{1}{4}\right)\lambda R}{a}$$

minima:

$$a \sin\theta + \frac{\lambda}{4} = \left(n + \frac{1}{2}\right)\lambda$$

$$a \sin\theta = \left(n + \frac{3}{4}\right)\lambda$$

$$a \frac{y}{R} = \left(n + \frac{3}{4}\right)\lambda$$

$$y = \frac{\left(n + \frac{3}{4}\right)\lambda}{a}$$

HOMEWORK 5

3 $x = A \sin \omega t$

$$v = \frac{dx}{dt} = \omega A \cos \omega t$$

$$E = \frac{1}{2} m A^2 \omega^2$$

$$dx = \omega A \cos \omega t \, dt$$

Wilson - sommerfeld quantization:

$$\oint p \, dq = nh$$

$$\oint mv \, dx = nh$$

$$\oint m A\omega \cos \omega t (\omega A \cos \omega t) \, dt = nh$$

$$mA^2 \omega^2 \oint \cos^2 \omega t \, dt = nh$$

$$\theta = \omega t$$

$$t = \frac{\theta}{\omega}$$

$$dt = \frac{1}{\omega} d\theta$$

$$\frac{mA^2\omega^2}{\omega} \oint \cos^2\theta \; d\theta = nh$$

$$\frac{2E}{\omega} \oint \cos^2\theta \; d\theta = nh$$

$$\frac{2E}{\omega} \cancel{\oint} \left[\frac{1}{2}\theta\right]_0^{2\pi} = nh$$

$$\frac{2\pi E}{\omega} = nh$$

$$E = \frac{nh\omega}{2\pi}$$

$$\boxed{E = n\hbar\omega}$$

5-3 $$E_k = eV_0 = \frac{p^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$V_0 = \frac{1}{e} \cdot \frac{(1240\ eV.nm)^2}{2(5.11 \times 10^5\ eV)\ (0.04\ nm)^2}$$

$$= 940\ V$$

5-7 (a) If there is a node at each wall, then

$$n\left(\frac{\lambda}{2}\right) = L \quad \text{where} \quad n = 1, 2, 3, \ldots$$

or $\lambda = \frac{2L}{n}$

(b) $$p = \frac{h}{\lambda} = \frac{hn}{2L}$$

$$E = \frac{p^2}{2m} = \frac{(hn/2L)^2}{2m} = \frac{h^2 n^2}{8mL^2}$$

$$E_n = \frac{(hc)^2 n^2}{8mc^2 L^2}$$

For $n=1$: $E_1 = \dfrac{(1240\ eV \cdot nm)^2\ (1)^2}{8\ (938 \times 10^6\ eV)\ (0.01 nm)^2}$

$= 2.05\ eV$

For $n=2$: $E_2 = 2.05\ eV\ (2)^2 = 8.20\ eV$

5-11. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2 m_p E_k}} = 0.25\ nm$

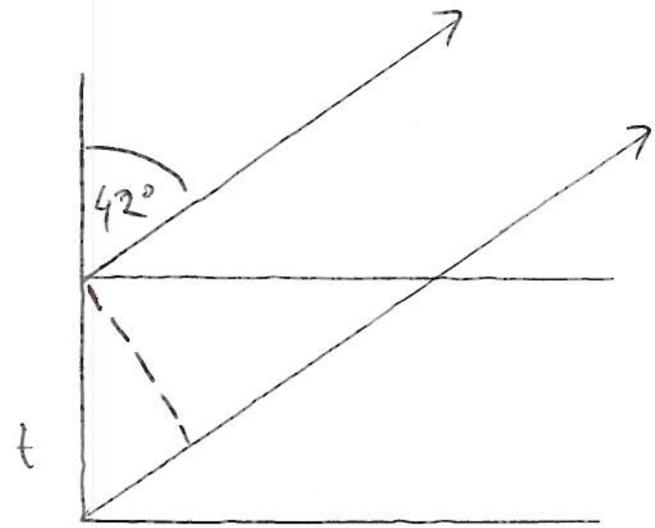
$$E_k = \frac{h^2}{2 m_p \lambda^2} = \frac{(hc)^2}{2(m_p c^2)\lambda^2}$$

$$= \frac{(1240\ eV \cdot nm)^2}{2\ (938 \times 10^6\ eV)\ (0.25\ nm)^2} = 0.013\ eV$$

$n\lambda = D \sin\phi \rightarrow \sin\phi = \dfrac{n\lambda}{D} = \dfrac{(1)\ (0.25\ nm)}{(0.304\ nm)}$

$\sin\phi = 0.822 \rightarrow \phi = 55°$

5-13



$$d = t \cos 42°$$

$$n\lambda = t + d = t(1 + \cos 42°)$$
$$= 0.30 \text{ nm } (1 + \cos 42°)$$

For the first maximum $n = 1$, so $\lambda = 0.523 \text{ nm}$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \rightarrow E_k = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$E_k = \frac{(1240 \text{ eV.nm})^2}{2(939 \times 10^6 \text{ eV})(0.523 \text{ nm})} = 3.0 \times 10^{-3} \text{ eV}$$

5-17 (a) $y = y_1 + y_2$

$$= 0.002\,m \cos\left(\frac{8.0x}{m} - \frac{400t}{s}\right) + 0.002\,m \cos\left(\frac{7.6x}{m} - \frac{380t}{s}\right)$$

$$= 2(0.002\,m)\cos\left[\frac{1}{2}\left(\frac{8.0x}{m} - \frac{7.6x}{m}\right) - \frac{1}{2}\left(\frac{400t}{s} + \frac{380t}{s}\right)\right]$$

$$\times \cos\left[\frac{1}{2}\left(\frac{8.0x}{m} + \frac{7.6x}{m}\right) - \frac{1}{2}\left(\frac{400t}{s} + \frac{380t}{s}\right)\right]$$

$$= 0.004\,m \cos\left(\frac{0.2x}{m} - \frac{10t}{s}\right) \times \cos\left(\frac{7.8x}{m} - \frac{390t}{s}\right)$$

(b) $v = \frac{\bar{\omega}}{k} = \frac{390/s}{7.8/m} = 50\ m/s$

(c) $v_s = \dfrac{\Delta \omega}{\Delta k} = \dfrac{20/s}{0.4/m} = 50\ m/s$

(d) Successive zeros of the envelope requires that

$$\frac{0.2\ \Delta x}{m} = \pi \text{ , thus}$$

$$\Delta x = \frac{\pi}{0.2} = 5\pi\ m$$

with $\Delta k = k_1 - k_2 = 0.4\ m^{-1}$

and $\Delta x = \dfrac{2\pi}{\Delta k} = 5\pi\ m$

5-22 (a) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}} = \dfrac{hc}{\sqrt{2mc^2 E_k}} = \dfrac{1240\ eV \cdot nm}{[2(0.511 \times 10^6 eV)(5eV)]^{\frac{1}{2}}}$

$= 0.549\ nm$

$d \sin\theta = \dfrac{\lambda}{2}$ For first minimum

$d = \dfrac{\lambda}{2\sin\theta} = \dfrac{0.549\ nm}{2\ \sin 5°} = 3.15\ nm$ slit separation

(b) $\sin 5° = \dfrac{0.5\ cm}{L}$ where L = distance to detector plane

$L = \dfrac{0.5\ cm}{2\ \sin 5°} = 5.74\ cm$