Homework #8 (Due 11/19/08)

From Streetman:

5.11, 5.12, 5.19, 5.22

5. Imagine a Si pn junction with Na=10^{17}cm^{-3} and Nd=2*10^{17}cm^{-3} at 300K. The junction has a cross-sectional area of 100μm^2, and the thickness of the p-type material is 2μm, and the n-type 3μm. $\tau_p = \tau_n = 1\mu s$ and $\mu_n = 1350$ cm^2/Vs, $\mu_p =$ 480 cm^2/Vs.

a) What is Vo and W at equilibrium?

b) Assume the potential at the junction is decreased by 0.4eV by the application of an external bias. What is I?

c) What is the total applied bias for the situation in (b)?
1) Assume W=0
2) Assume $\varepsilon(x)$ (electric field) is constant (but not necessarily the same) in the p- and n- type material.

d) Are assumptions 1 and 2 from part (c) reasonable? Explain why or why not.

e) What happens when V_f>>V_o? Can you have a situation where the potential drop across the junction becomes negative? Use your results from a)-d) to explain.

6. Imagine a GaAs pn-junction with N_a=10^{16}cm^{-3} and N_d=5*10^{16}cm^{-3}, at 300K, with cross-sectional area A.
a) Calculate W and Vo at equilibrium.
b) What is the total charge of the p- and n- sides of the depletion region at equilibrium? If a reverse bias (Vr) is applied to the junction, what are the new values for the charges stored in the depletion region?

5.11 p+n $N_d = 10^{16}$

$n_i = 10^{10}\ cm^{-3}$ $\epsilon_r = 12$ $D_n = 50\ cm^2/s$

$D_p = 20\ cm^2/s$

$\tau_n = 100ns$

$\tau_p = 50ns$

$V_F = 0.6V$

CALCULATE $J_{p\,diff}\ (x_n = 2\mu m)$

$$J_{p\,diff} = -q D_p \frac{d\,\delta p(x_n)}{\partial x_n}$$

$n_n = 10^{16}\ cm^{-3}$ $\quad p_n = \frac{n_i^2}{n_n} = \frac{10^{20}}{10^{16}} = 10^4\ cm^{-3}$

$$I = qA\left(\frac{D_p}{L_p} p_n + \cancel{\frac{D_n}{L_n} n_p}\right)\left(e^{qV/kT} - 1\right)$$

$n_p \approx 0$

$$I_{diff} = qA \frac{D_p}{L_p} p_n e^{qV/kT}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{20 \times 50 \times 10^{-9}} = 0.001\ cm$$

$$I_p(x_n) = \frac{qAD_p}{L_p} \Delta p_n e^{-x/L_p} \qquad \Delta p_n = p_n e^{qV/kT}$$

$$= \frac{qAD_p}{L_p} p_n e^{qV/kT} e^{-x/L_p}$$

5.11 cont.

$$J_p(x_n) = \frac{q D_p}{L_p} p_n e^{qV/kT} e^{-x/L_p}$$

$$= 3.2 \times 10^{-11} e^{qV/kT} e^{-x/L_p}$$

ASSUME T = 300K

$$= 0.368 e^{-x/L_p}$$

for $x = 2\mu m = 2 \times 10^{-4}$ cm

$L_p = 10\mu m$

$x/L_p = 0.2$

$$\boxed{J_p(x_n = 2\mu m) = 0.301\ A}$$

If p^+ is doubled, what effect on $J_p(x_n = 2\mu m)$?

BASICALLY NO EFFECT! $J_{p\,diff} \propto p_n$ CHANGING P_p doesn't affect p_n. WILL SHIFT POSITION OF x_{no}, but this doesn't matter, since we are measured in $x_n = 2\mu m$.

5.12 1

$p^+n \Rightarrow N_d = 5\times10^{16}\ cm^{-3}$ $\quad A = 10^{-3}\ cm^2$

$\tau_p = 1\mu s \quad D_p = 10\ cm^2/s$

What is I for $V_f = 0.5V$ @ 300K?

$$I = qA\left(\frac{D_p}{L_p}p_n + \cancel{\frac{D_n}{L_n}n_p}\right)\left(e^{qV/kT} - 1\right)$$

$$n_p \approx 0$$

$$I = qA\frac{D_n}{L_p}p_n\left(e^{qV/kT} - 1\right)$$

$$n_n = 5\times10^{16}$$

$$p_n = \frac{n_i^2}{5\times10^{16}} = \frac{2.25\times10^{20}}{5\times10^{16}} = 4500$$

$$\frac{I}{A}(0.5V) = 1.6\times10^{-19}\cdot10^{-3}\cdot\frac{10}{L_p}\left(e^{0.5/.0259} - 1\right)$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10\times1\times10^{-6}}$$

$$L_p = 0.00316\ cm$$

$$I(0.5V) = 5.05\times10^{-19}\left(e^{0.5/.0259} - 1\right)$$

$$\boxed{I = 5.953\times10^{-7}\ A}$$

5.19

Si p-n junction

$A = 0.001\ cm^2$ $N_a = 10^{15}\ cm^{-3}$ $N_d = 10^{17}\ cm^{-3}$

a) V_0

$$\frac{KT}{q} \ln \frac{n_n}{n_p} = V_0$$

$n_n = 10^{17}$ $n_p = \frac{n_i^2}{P_p} = \frac{2.25\times10^{20}}{10^{15}} = 2.25\times10^5$

$$V_0 = \frac{KT}{q} \ln \frac{10^{17}}{2.25\times10^5}$$

$$\boxed{V_0 = 0.694\ eV}$$

b) $W\ (V=0)$

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

$\epsilon = 11.8 \times 8.85\times10^{-14}\ F/cm$

$W = 9.56\times10^{-5}\ cm$

$$\boxed{W = 0.956\ \mu m}$$

5.19 cont'd

c)

$I\ (V = 0.5V)$

$\mu_n = 1500\ cm^2/V\text{-}s \quad \mu_p = 450\ cm^2/V\text{-}s$

$\tau_n = \tau_p = 2.5\ ms$

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT} \cancel{-1}\right)$$

↑ still dominant

$n_p = 2.25 \times 10^5\ cm^{-3}$

$$p_n = \frac{2.25\times10^{20}}{n_n} = \frac{2.25\times10^{20}}{10^{17}} = 2.25\times10^{3}$$

$D_p = \mu_p \frac{kT}{q} = 11.64\ cm^2/s$

$D_n = \mu_n\ kT/q = 38.81$

$L_p = \sqrt{D_p\tau_p} = 0.17\ cm$

$L_n = \sqrt{D_n\tau_n} = 0.311\ cm$

$$I = 1.6\times10^{-22}\left(11.54\times10^{5} + 2.81\times10^{7}\right)\cdot 2.47\times10^{8}$$

$$\boxed{I = 1.114\times10^{-6}\,A}$$

Current is carried mostly by electrons in p-type region.

To double electron current, cut p-doping of p-type region in half.

5.22

Si p-type doping 2×10^{16} cm^{-3} LEFT

p-type doping 10^{18} cm^{-3} RIGHT

at 600K, $n_i = 10^{16}$ cm^{-3}

LEFT

CHARGE NEUTRALITY MEANS:

$$p - n + N_d^+ - N_a^- = 0$$

$$N_d = 0$$

$$p - n - N_a^- = 0$$

$$N_a^- = 2\times10^{16}$$

$$n = \frac{n_i^2}{p}$$

$$p - \frac{n_i^2}{p} - N_a^- = 0$$

$$p^2 - N_a^- p - n_i^2 = 0$$

$$\frac{N_a^- \pm \sqrt{N_a^2 + 4(1)(n_i^2)}}{2} = \frac{2\times10^{16} \pm \sqrt{4\times10^{32} + 4\times10^{32}}}{2}$$

$$\boxed{p_{LEFT} = 2.414\times10^{16}\ cm^{-3}}$$

$$p_{RIGHT} = \left(10^{18} \pm \sqrt{10^{36} + 4\cdot10^{32}}\right)/2$$

$$\boxed{p_{right} = 10^{18}}$$

5-22
cont.

$p_{left} = n_i e^{-(E_F - E_i)/kT}$ kT = 600 K

$p_{right} = n_i e^{(E_i - E_F)/kT}$

$-(E_F - E_i) = kT \ln \frac{p_{left}}{n_i}$

$E_i - E_F = kT \ln \frac{p_{right}}{n_i}$

$E_i - E_F = 0.0456$

$E_i - E_F = 0.238\ eV$

$\boxed{V_0 = \Delta E_i = 192.7\ meV}$

5

Si pn junction

$N_a = 10^{17}$ $N_d = 2\times10^{17}$ 300K

$A = 100\mu m^2$

thickness p = $2\mu m$
thickness n = $3\mu m$

$\tau_p = \tau_n = 1\mu s$ $\mu_n = 1350$ $\mu_p = 480$

a) What is V_0?

$$V_0 = \frac{KT}{q} \ln \frac{p_p}{p_n}$$

$p_p = 10^{17}$ $n_p = \frac{2.25\times10^{20}}{1\times10^{17}}$

$p_n = \frac{2.25\times10^{20}}{2\times10^{17}} = 1125$ $n_p = 2.25\times10^{3}$

$$V_0 = .025875 \ln \frac{10^{17}}{1125} = 0.83106 \text{ eV}$$

$$W = \left[\frac{2\epsilon V_0}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)\right]^{1/2}$$

$$= 1.3993\times10^{-5} cm = 0.13\mu m$$

b) $$I = qA\left(\frac{D_p}{L_p} p_n + \frac{D_n}{L_n} n_p\right)\left(e^{qV/KT} - 1\right)$$

$D_p = \frac{KT}{q}\mu_p = 12.42$ $L_p = \sqrt{D_p\tau_p} = 0.00352$

$D_n = \frac{KT}{q}\mu_n = 34.93$

$L_n = \sqrt{D_n\tau_n} = .00591$

5 (cont)

$$I = 1.6\times10^{-25}\left(\frac{12.42 \cdot .1125}{.00352} + \frac{34.93 \cdot 2250}{.00591}\right)\left(e^{qV/kT} - 1\right)$$

$$\boxed{I = 1.428\times10^{-11}\ A}$$

c)

$$R_p = \frac{l_p}{wt}\frac{1}{\sigma} = \frac{l_p}{A}\frac{1}{\sigma_p}$$

$$\sigma_p = q\mu_p N_p$$

$$= 1.6\times10^{-19} \cdot 10^{17} \cdot 480$$

$$R_p = \frac{2\times10^{-4}}{1\times10^{-6}} \cdot \frac{1}{1.6\times10^{-19}\cdot10^{17}\cdot480} = 26\ \Omega$$

$$R_n = \frac{3\times10^{-4}}{1\times10^{-6}}\ \frac{1}{1.6\times10^{-19}\cdot 2\times10^{17}\cdot1350} = 6.9\ \Omega$$

$$V_n = IR_n = 9.92\times10^{-11}\ V$$

$$V_p = IR_p = 3.7\times10^{-10}\ V$$

d) Yes.

1) $W \ll l_n + l_p$

$.13\mu m \ll 2 + 3\mu m$

2) Yes $l_n, l_p \gg L_n, L_p$

e) If $V_{applied} >> V_0$

Say $V_F = 0.8\ V$

$I(0.8V) = 7.39 \times 10^{-5}\ A$

$V_{n+p} = 2.4\ mV$

$I(1.2V) = 382\ A$

$V_{n+p} = 12.6\ kV$!!

$I(1V) = 0.168\ A$

$V_n + V_p = 5.54\ V$

$I(0.9V) = 3.5\ mA$

$V_n + V_p = 0.116\ V$

Technically, it could be possible that $V_{junction} > V_0$, but it is clear that when $V_{junction} \approx V_0$, need to drop a lot of V across resistors in diode → this will heat up diode, eventually melting it!

6) GaAs pn-junction $N_a = 10^{16}$ $N_d = 5\times10^{16}$ 300K

a)

$$W = \left[\frac{2\epsilon V_0}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

$$V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n}$$

$p_p = 10^{16}$

$p_n = 0.00008$

$\boxed{V_0 = 1.197\, eV}$

$$W = \left[\frac{2 \cdot 13.2\epsilon_0 \cdot 1.197}{1.6E-19} \left[\frac{1}{1E16} + \frac{1}{5E16} \right] \right]^{1/2}$$

$= 4.18 \times 10^{-5}\ \text{cm} = 0.418\ \mu m$

b) At equilibrium

$$|Q_p| = |Q_n| = qAN_d x_{no} = qAN_a x_{po}$$

for $V = V_r$ $x_{po} = \frac{W}{1 + N_a/N_d} = \frac{0.458\,\mu m}{1 + 1/5} = 0.3816\,\mu m$

charge stored in depletion ... O

$\boxed{Q = A \cdot 6.1 \times 10^{-18}\ C}$ cm

for $V_r = V$ $Q = \frac{qAN_a}{1 + \frac{N_a}{N_d}} \cdot W(v) \cong 4.2 \times 10^{-5} \sqrt{V_r}$ cm