MIDTERM EXAM #1

1)

$$g(\epsilon) = \frac{m^* d\epsilon}{\pi \hbar^2} \qquad m^* = 0.067 m_0$$

a) $\epsilon_c - \epsilon_F = 0.1 eV$

What are ϵ_1, ϵ_2?

For an infinite QW

$$E(n) = \frac{\hbar^2 \pi^2 n^2}{2ma^2} = 8.91 \times 10^{-21} J$$

$$E_n = 55.7 \cdot n^2 \text{ meV}$$

$$\boxed{\begin{aligned} \epsilon_1 &= 55.7 \text{ meV} \\ \epsilon_2 &= 223 \text{ meV} \end{aligned}}$$

$$n_n(\epsilon) = \int_{\epsilon_n}^{\infty} g(\epsilon) f(\epsilon)\, d\epsilon$$

$$= \frac{m^*}{\pi \hbar^2} \int_{\epsilon_n}^{\infty} \frac{1}{e^{(\epsilon-\epsilon_F)/kT} + 1}\, d\epsilon \approx \frac{m^*}{\pi \hbar^2} \int_{\epsilon_n}^{\infty} e^{-(\epsilon-\epsilon_F)/kT}$$

$$n_n(\epsilon) = -\frac{m^*}{\pi \hbar^2} e^{\epsilon_F/kT} kT \left[e^{-\epsilon/kT} \right]_{\epsilon_n}^{\infty}$$

$$= \frac{m^*}{\pi \hbar^2} kT\, e^{-(\epsilon_n - \epsilon_F)/kT}$$

n	$\epsilon_n - \epsilon_F$	77K	300K
1	$E_1 - E_F = 155.7$ meV	$1.23 \times 10^5\ m^{-2}$	$1.78 \times 10^{13}\ m^{-2}$
2	$E_2 - E_F = 323$ meV	$1.41 \times 10^6\ m^{-2}$	$2.76 \times 10^{10}\ m^{-2}$

c) For 99% of carriers in 1st Energy level

$$\frac{n_1}{n_2} = 99 = \frac{\cancel{\frac{m^*}{\pi\hbar^2} kT}\, e^{-(E_1 - E_F)/kT}}{\cancel{\frac{m^*}{\pi\hbar^2} kT}\, e^{-(E_2 + E_F)/kT}}$$

$$99 = e^{-(E_1 - E_F)/kT + (E_2 - E_F)/kT}$$

$$99 = e^{(E_2 - E_1)/kT}$$

$$kT \ln 99 = E_2 - E_1$$

at room T

at 77K

$26\,meV \cdot 4.595 = E_2 - E_1$

$6.6\,meV \cdot 4.595$

$\Delta E_{21}(300K) = .119\,eV$

$\Delta E_{21}(77K) = 30.5\,meV$

$$\Delta E_{21} = \frac{\hbar^2 \pi^2}{2m^*}(2^2 - 1^2)\, a^{-2} = \frac{3\hbar^2\pi^2}{2m^*}\, a^{-2}$$

$$= 1.67 \times 10^{-17}\, a^{-2}$$

$$a\,(300K) = \left(\frac{.119}{1.67 \times 10^{-17}}\right)^{-1/2}$$

$$a\,(77K) = \sqrt{\frac{1.67 \times 10^{-17}}{.0305}}$$

$\boxed{a(300K) = 11.8\,nm}$

$\boxed{a(77K) = 23.4\,nm}$

2)

BCC w/ $a = 5\AA$

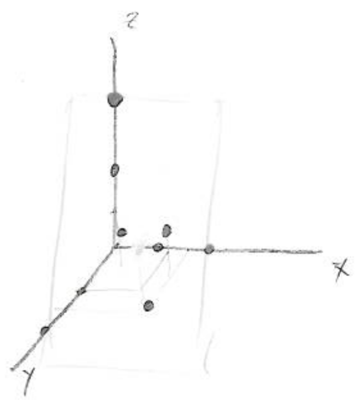
a) PVs for BCC

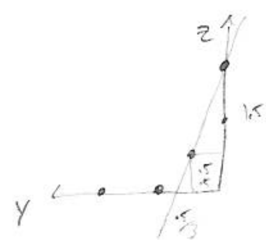
$$\vec{r}_1 = a\hat{x} \qquad \vec{r}_2 = a\hat{y} \qquad \vec{r}_3 = \tfrac{a}{2}(\hat{x} + \hat{y} + \hat{z})$$

To get to $\vec{R} = \frac{3a}{2}(\hat{x} + \hat{y} + \hat{z})$

Simply $= \boxed{3\vec{r}_3}$

b)





Intercepts
z @ 2
y @ $2/3$
x @ ∞

Take reciprocal of intercepts

$$(\infty, \tfrac{2}{3}, 2) \longrightarrow (0, \tfrac{3}{2}, \tfrac{1}{2})$$

Find smallest integer representation

$$\boxed{(0\ 3\ 1)}$$

c) Simple cubic $\vec{r}_1 = a\hat{x} \quad \vec{r}_2 = a\hat{y} \quad \vec{r}_3 = a\hat{z}$

Basis $\rightarrow \vec{0},\ \tfrac{a}{2}(\hat{x} + \hat{y} + \hat{z})$

3)

a)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\,\psi = E\psi$$

$$V(x) = \begin{cases} \infty & x<0 \\ 0 & 0<x<a \\ V_0 & x>a \end{cases}$$

b)

I $(x<0)$ $\quad \psi_1 = 0$

II $(0<x<a)$ $\quad \psi_2 = A\sin k_2 x + B\cos k_2 x$

III $(x>a)$ $\quad \psi_3 = Ce^{-k_3 x} + De^{k_3 x}$

c) BOUNDARY CONDITIONS

$$\psi_1(0) = \psi_2(0)$$

$$\psi_2(a) = \psi_3(a) \quad \text{AND} \quad \psi_3(\infty) = 0$$

$$\psi_2'(a) = \psi_3'(a)$$

d)

$$\psi_2 = A \sin k_2 x \qquad k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\psi_3 = C e^{-k_3 x} \qquad k_3 = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

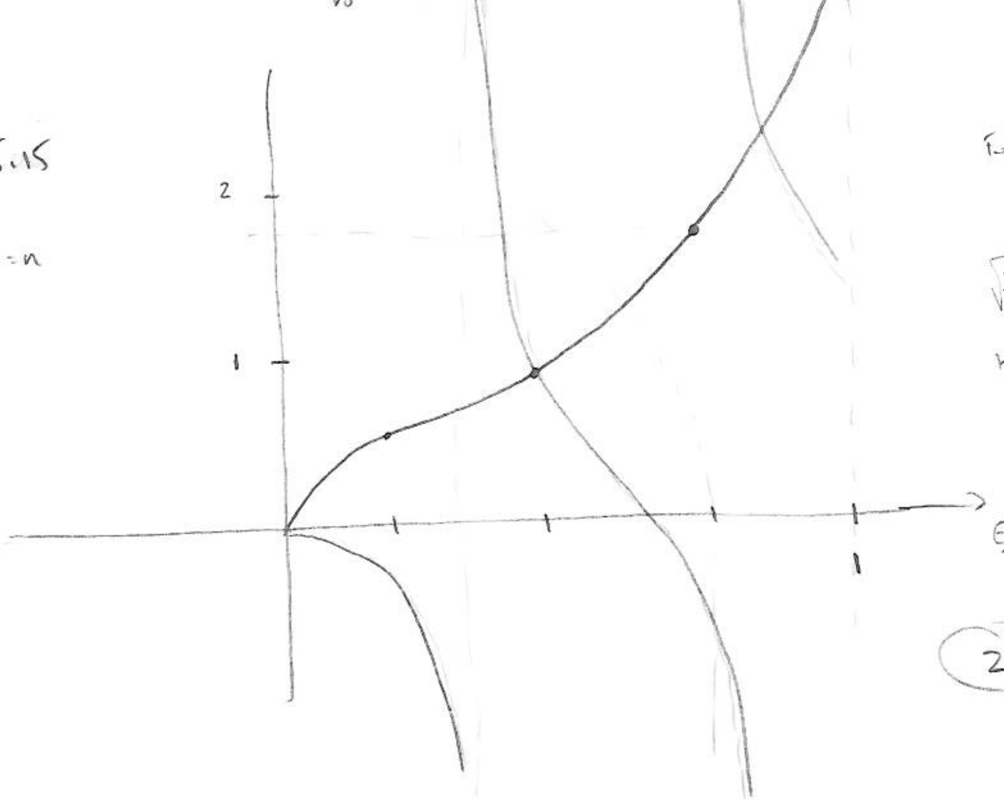
① $A \sin k_2 a = C e^{-k_3 a}$ ② $A k_2 \cos k_2 a = -C k_3 e^{-k_3 a}$

$$\frac{①}{②} = \frac{1}{k_2} \tan k_2 a = -\frac{1}{k_3}$$

$$\tan k_2 a = \frac{-k_2}{k_3}$$

$$\boxed{-\tan k_2 a = \frac{k_2}{k_3}} \qquad \frac{k_2}{k_3} = \sqrt{\frac{\frac{2\cancel{m}E}{\cancel{\hbar^2}}}{\frac{2\cancel{m}(V_0-E)}{\cancel{\hbar^2}}}} = \sqrt{\frac{E}{V_0-E}}$$

$$\frac{k_2}{k_3} = \sqrt{\frac{E/V_0}{1-E/V_0}}$$



$k_2 a\,(E=V_0) = 5.15$

$\frac{k_2 a}{\pi} = 1.64 = n$

$-\tan k_2 a \to 0$'s @ $k_2 a = n\pi$

$-\infty$'s @ $k_2 a = \frac{\pi}{2} + 2\pi n$

∞'s @ $k_2 a = \frac{3\pi}{2}$

for 0's ⇒

$k_2 = \frac{n\pi}{a}$

$\sqrt{\frac{2mE}{\hbar^2}} = \frac{n\pi}{a}$

How many 0's up to V_0?

$E = V_0 \quad a = 1\,nm$

4)

$$E_g = 1.2\,eV$$
$$E_i - E_v = 0.5\,eV$$

a) What is $\frac{m_n^*}{m_p^*}$?

$$n_i = N_c e^{-(E_c - E_i)/KT} = p_i = N_v e^{-(E_i - E_v)/KT}$$

$$\frac{N_c}{N_v} = \frac{e^{-(E_i - E_v)/KT}}{e^{-(E_c - E_i)/KT}}$$

N_c, N_v differ only in $(m^*)^{3/2}$ term

$$\left(\frac{m_n^*}{m_p^*}\right)^{3/2} = e^{-(E_i - E_v)/KT + (E_c - E_i)/KT}$$

$$\left(\frac{m_n^*}{m_p^*}\right)^{3/2} = e^{-(0.5)/KT + (0.7)/KT}$$

$$\left(\frac{m_n^*}{m_p^*}\right)^{3/2} = e^{0.2/KT} = 2257.5$$

$$\boxed{\frac{m_n^*}{m_p^*} = 172.1}$$

b) If $m_n^* = 0.8$

$$m_p^* = \frac{m_n^*}{172.1}$$

$$n_i = \sqrt{N_c N_v}\, e^{-E_g/2kT} \quad \text{or} \quad n_o = N_c\, e^{-(E_c - E_F)/kT}$$

$$\text{or} \quad p_o = N_v\, e^{-(E_F - E_v)/kT}$$

$$n_o = \underbrace{2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}}_{N_c} e^{-(E_c - E_i)/kT}$$

$$n_o = \underset{3.262}{\cancel{4.613}} \times 10^{13}\ m^{-3} = \underset{3.262}{\cancel{4.61}} \times 10^{7}\ cm^{-3}$$

$$p_o = \underset{3.262}{\cancel{4.61}}3 \times 10^{13}\ m^{-3} = \underset{3.262}{\cancel{4.61}} \times 10^{7}\ cm^{-3}$$

$$\boxed{n_i = \underset{3.262}{\cancel{4.61}} \times 10^{7}\ cm^{-3}}$$

c) $N_A = 5 \times 10^{17}\ cm^{-3}$

$p_o = 3.75 \times 10^{17}\ cm^{-3}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(\underset{3.262}{\cancel{4.61}} \times 10^{7})^3}{3.75 \times 10^{17}} = \underset{0.002837}{\cancel{0.0056}}\ cm^{-3}$$

5)

$L_x = 1nm \qquad L_y = 2nm \qquad L_z \gg L_x, L_y$

a)

$$\boxed{E_{n_x, n_y, k_z} = \frac{\hbar^2 \pi^2}{2m^*}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right) + \frac{\hbar^2 k_z^2}{2m^*}}$$

b)

Linear volume / state $= \pi / L_z$

$$\# \text{ states in } dK = \underset{\text{spin}}{\underset{\uparrow}{2}} \cdot \frac{dK}{\pi / L_z}$$

$$E_z = \frac{\hbar^2 k^2}{2m^*}$$

$$\partial E / \partial K = \frac{2\hbar^2 k}{2m^*}$$

$$dK = \frac{2m^* \, dE}{2\hbar^2 K} \qquad K = \sqrt{\frac{2m^* E}{\hbar^2}}$$

$$dK = \frac{m^* \, dE}{\hbar^2 \sqrt{\frac{2m^* E}{\hbar^2}}} = \frac{\sqrt{m^*} \, dE}{\hbar \sqrt{2} \sqrt{E}} = \sqrt{\frac{m^*}{2\hbar^2}} \frac{dE}{\sqrt{E}}$$

$$\# \text{ states in } dE = \frac{2\sqrt{\frac{m^*}{2\hbar^2}} \frac{dE}{\sqrt{E}}}{\pi / L_z}$$

b) cont

$$\frac{\#\text{ states in } dE}{V} = \boxed{g(E) = \sqrt{\frac{2m^*}{\hbar^2 \pi^2 E}}\; dE}$$

c)