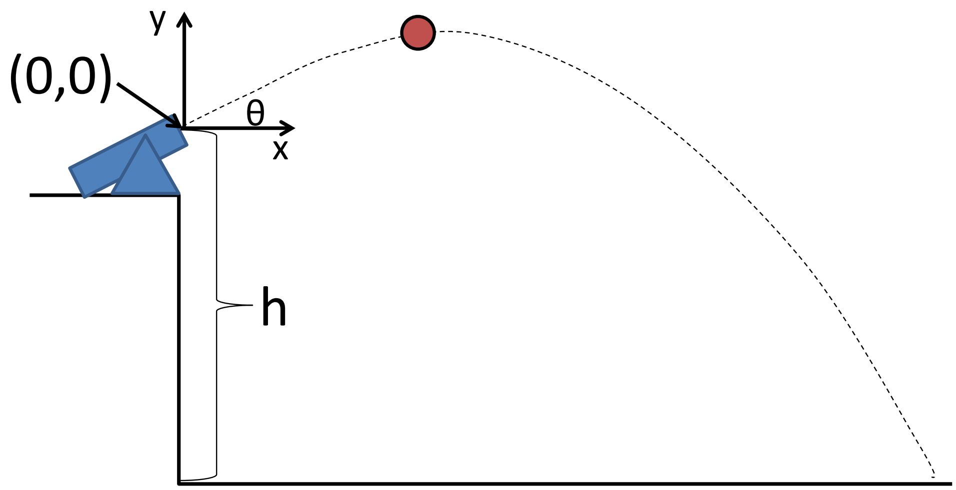
Problem 1: A cannonball is shot from cannon at the edge of a cliff (h=100), as shown in the figure below. The cannonball is shot at an angle θ=30°. Use the coordinate system shown for this problem.



(a) The cannonball reaches a maximum height of y=30m. What is the y-component of the initial velocity?

v_{yo} =_______________

(b) What is the x-component of the cannonball's initial velocity?

v_{xo} =_______________

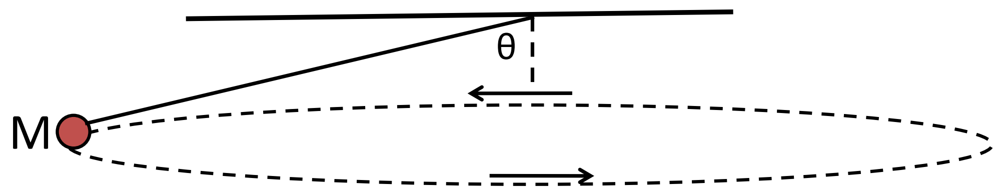
(c) How long does it take for the cannon ball to get to its maximum height?

t =____________

(d) Where does the cannonball land?

Landing Position=_(______,_______)_

Problem 2 (25 points): A mass (M=3kg) is attached to a weightless cord of length L=2m is spun in a circular path. θ=85°.



a) (5 pts) Draw the free body diagram for the mass at the point in time shown in the figure above.

b) (5 pts) What is the tension in the cord?

F_T=____________

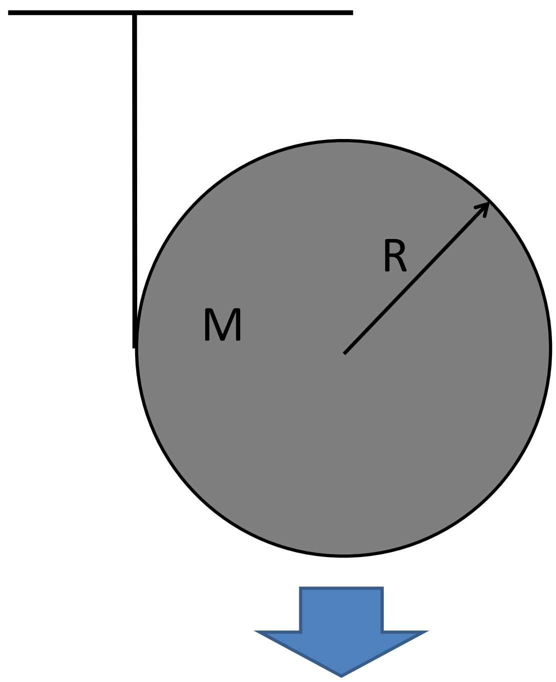
c) (10 pts) What is the tangential velocity of the mass?

v_{tan}=____________

d) (5pts) How long does it take the mass to make one full rotation?

T=____________

Problem 3: A circular disk (yo-yo) (R=5cm) is released from rest and rolls without slipping down a massless cord fixed at its top end. The motion of the yo-yo consists of the translational motion of the center of mass and rotational motion about the center of mass. $I_{yoyo}=MR^2/2$.



a) Draw the free body diagram for the yo-yo.

b) Write out Newton's second law for the rotational motion of the yo-yo about the center of mass of the yo-yo.

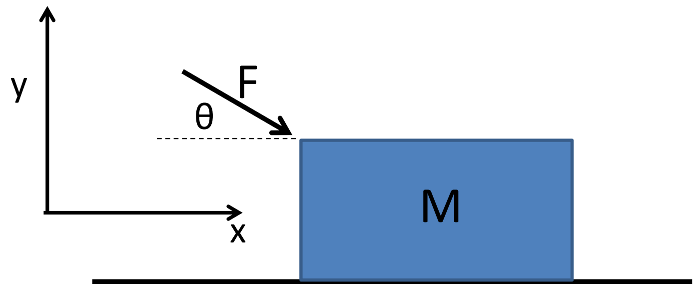
c) Write out Newton's second law for the translational motion of the center of mass of the yo-yo.

d) Determine the translational acceleration of the yo-yo's center of mass and the angular acceleration of the yo-yo about its center of mass.

$a =$ ______________

$\alpha =$ ______________

Problem 4: A 10kg block sits on a surface with μ_s=0.6 and μ_k=0.4. You exert force (F) at an angle of θ=25°on this block.



a) Draw the free body diagram for the block.

b) What is the minimum Force with which you can push the block and have it move?

F_{min}=______________

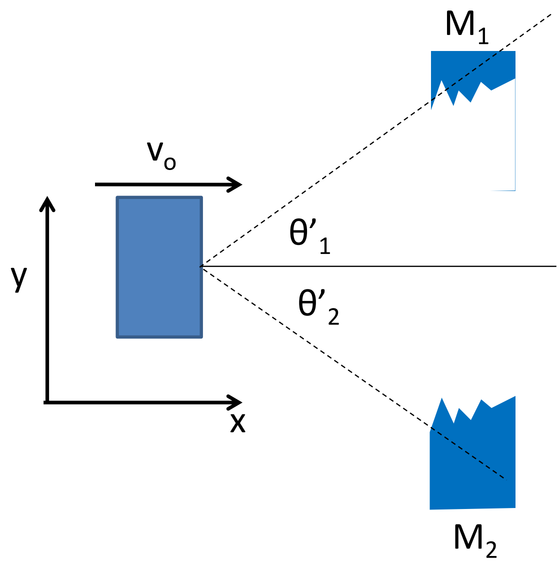
c) You push the block with the Force determined in part (b) and it begins to move. What will the acceleration of the block be?

a=______________

d) (5pts) You apply this Force for 10s. What is the net work done on the block over this time?

W_{net}=______________

Problem 5 (25 points): A block of mass M=10kg slides across a frictionless surface with a speed of v_o=20 m/s, as shown below. A small explosive on the block is detonated remotely, and the block is broken into two pieces (M_1=3kg and M_2=7kg). M_1 leaves the explosion with a speed of 23m/s at an angle of 34°.



a) (10 pts) What is the speed of M_2 after the explosion?

$v_2' =$ ____________

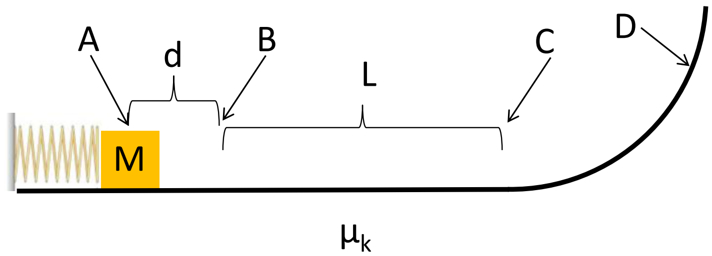
b) (10 pts) At what angle does M2 leave the collision? Hint: Check your answer carefully! Remember, if $\sin(\theta)=0.5$, θ can be either 30° or 150°!!!

$\theta_2' =$ ____________

c) (5 pts) What is the minimal energy of the explosion? Hint: How much did the kinetic energy of the system change by?

$E_{explosion} =$ ____________

Problem 6 (25 points): A mass (M=3kg) is placed in front of a spring with k=1200N/m, compressed d=40cm. The spring is released, shooting the mass forward from A to B (frictionless). The mass then travels along a flat surface from B to C (L=20m), with μ_k=0.1. At C the surface becomes frictionless and smoothly inclines upwards. The speed of the mass at point D is 0m/s.



a) (5 pts) What is the velocity of the mass at B?

$v_B =$______________

b) (5 pts) What is the velocity of the mass at point C?

$v_C =$______________

c) (5pts) How high is point D above the flat surface?

H_D =______________

d) (10pts) Does the mass make it back to the spring? No credit for simple yes or no answer, must explain why or why not!

Problem 7 : A car travels around a curve of radius R=100m at a constant speed of 30m/s :

(a) What is the centripetal acceleration of the car?

(b) If the curve is flat, and we can assume friction, draw the free body diagram for the car (show the coordinate system you choose).

(c) Determine the coefficient of friction required to keep the car from slipping. Is this a kinetic or static coefficient?

d) Now assume the road is covered in a thin sheet of ice, and is effectively frictionless. The curve is now banked. Draw the free body diagram for the car (show the coordinate system you choose).

e) At what angle must the road be banked to prevent the car from slipping?

Problem 8: Working with vectors.

(a) A train is moving with a velocity $\vec{v}_{train} = 15\hat{i} + 68\hat{j}$ relative the Earth. A passenger on the train walks across the train with a velocity $\vec{v}_{passenger} = 2\hat{i} + 1\hat{j}$. What is the passenger's velocity relative to the Earth?

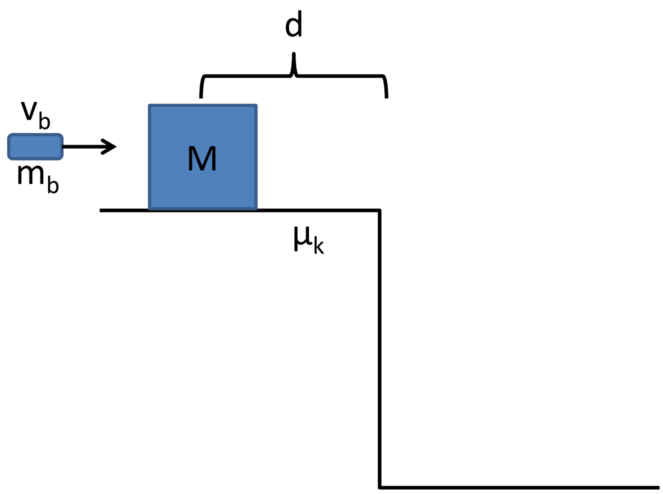
(b) A mass moves a distance $\vec{d}_{mass} = 30\hat{i} + 16\hat{j}$ while a Force $\vec{F} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ acts on the mass. How much work is done by the Force?

(c) What is the angle between the vectors $\vec{A} = 5\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$?

(d) A force $\vec{F} = -2\hat{i} + 5\hat{j} - \hat{k}$ is applied at a distance $\vec{r} = 3\hat{j} - 3\hat{k}$ from an object's center of mass. What torque is exerted by the Force about the object's center of mass?

(e) A mass of 3kg moves with a constant velocity $\vec{v} = 4\hat{i} + 3\hat{j} + 2\hat{k}$. At time T, the mass is at position $\vec{r} = \hat{j} + \hat{k}$. What is the angular momentum (about the origin) of the mass at time T?

Problem 9 : A mass (M=5kg) sits a distance d=20cm from the edge of a table with coefficient of friction μ_k=0.2. A bullet of mass m_b=50g is shot at the mass and embeds in the mass. In this problem we are trying to find the minimum bullet velocity required to knock the mass over the edge of the table.



a) Draw a free body diagram for the Mass once the bullet is embedded as it slides along the table.

b) What is the deceleration of the mass+bullet as it slides?

a=______________

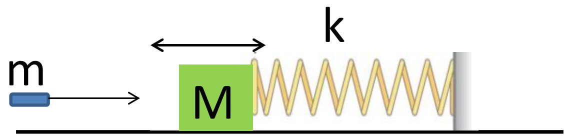
c) What is the minimum initial velocityof the bullet+mass required to move the mass a distance d?

v_o=_______________

d) What is the minimum velocity of the bullet needed to knock the mass+embedded bullet over the edge he table?

v_b=_______________

Problem 10 : A mass (M=5kg) is attached to a spring with spring constant k, and oscillates on a frictionless surface with an amplitude A.



a) If the equation of motion for mass M is $x(t) = 0.1\cos(1.5t + \pi/4)$, what is the spring constant of the spring? What is the amplitude A?

k=_____________

A=_____________

b) Where is the mass at time t=2s? What is the mass's velocity at this time?

x(t=2s)=_____________

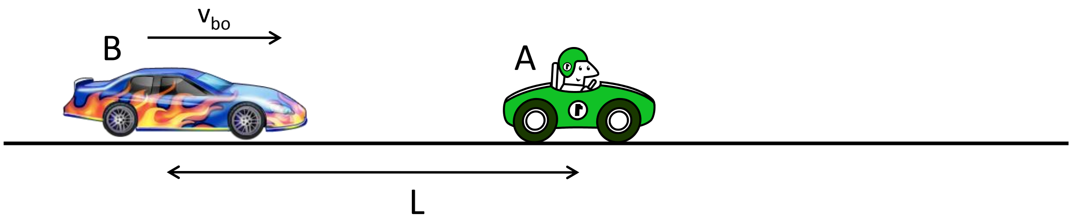
v(t=2s)=_____________

c) At time t=2s, a smaller mass (m=150g) is shot into the larger mass at a velocity of 200m/s and embeds in the larger mass. What is the velocity of the system immediately after the collision?

v'(t=2s)=____________

d) Give the new equation of motion for the system following the collision? Hint: you will have new amplitude, angular frequency, and phase, but you know both position and velocity at impact…..when you write out the new equations of motion, it makes sense to take the moment of impact as your new t=0.

Problem 11 : Two cars are racing. Car A starts L=15m ahead of Car B. Car B starts with an initial velocity of v_{bo}=5m/s, Car A starts from rest. Car A accelerates with a_A=2m/s^2, and Car B accelerates with a_B=1.5m/s^2.



a) Give the equations of motion for each car, assuming that x=0 refers to the starting point of Car B..

$x_A(t)$=_____________

$x_B(t)$=_____________

b) What are the positions and speeds of each car at t=2s?

$x_A(t=2)$=_____________

$x_B(t=2)$=_____________

$v_A(t=2)$=_____________

$v_B(t=2)$=_____________

c) Does Car B ever catch up with Car A? If so, when and where?

$x_{catch\text{-}up}$=______________

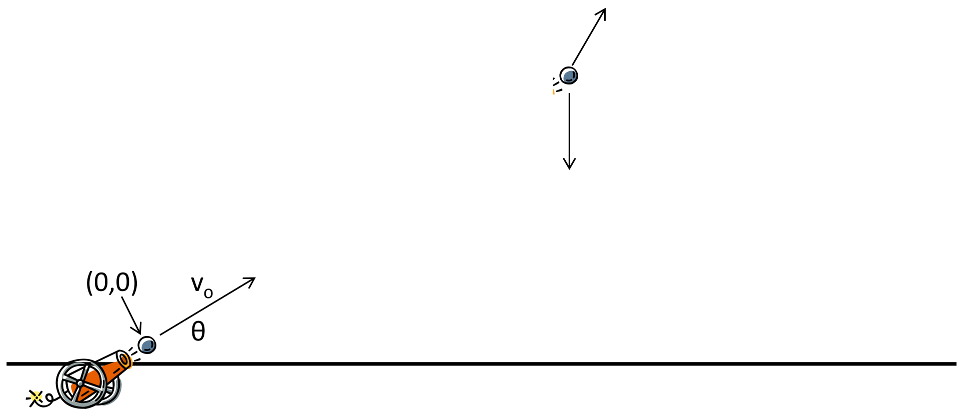
$t_{catch\text{-}up}$ =______________

d) If they race to a point a distance D from the origin, and it takes 20 seconds for the winner to get to D, who wins the race, and what is D?

winner=____________

D=____________

Problem 12 : A projectile (m=2kg) is shot from a cannon at an initial angle of θ=30° with an initial speed of v_o=40m/s. Assume the ball is shot from the origin.



a) Where is the peak of the object's trajectory? When is this point reached?

$\vec{r}_{peak}=$________________

$t_{peak}=$________________

b) At the peak of the object's trajectory, a charge in the object is detonated and the object splits into two equal masses. One half leaves the explosion with a velocity $\vec{v}_A=-10\hat{j}$. What is the velocity of the other half immediately after the collision?

$\vec{v}_B=$________________

c) Is Kinetic Energy lost, gained, or conserved in the explosion? Quantify your answer.

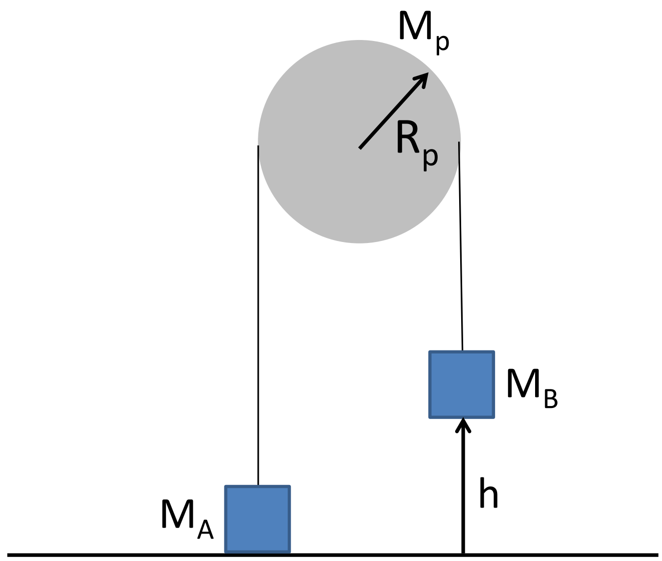
$\Delta KE =$ ______________

d) Where do the two masses land on the ground?

$\vec{r}_{Alanding} =$ ______________

$\vec{r}_{Blanding} =$ ______________

Problem 13 : Consider the Atwood machine shown below. The two masses are connected by a massless cord which is passed over a pulley of mass M_p and moment of inertia $I_{pulley} = 0.35 M_p R_p^2$. Mass B (8kg) starts a distance h=4m from the ground, and Mass A (4kg) starts just barely off the ground..



a) Draw the free body diagrams for the two masses and the pulley, including your coordinate systems.

b) The system is released from rest and it takes 4s for Mass B to hit the ground. what is the acceleration of the system?

a=______________

c) What are the tensions in the cords attached to Mass A and Mass B during the motion of the system?

T_A=____________

T_B=____________

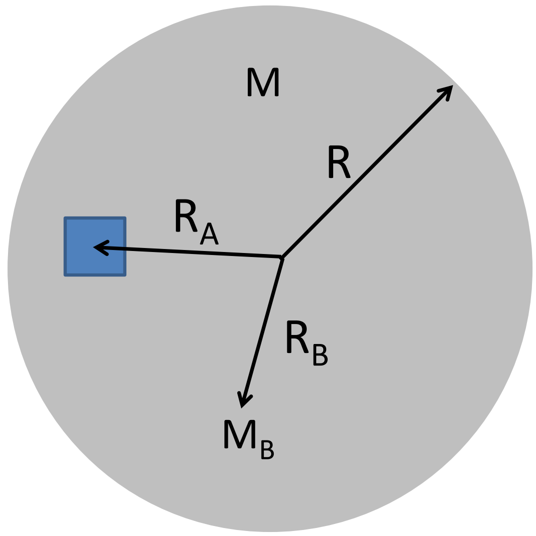
d) What is the mass of the pulley?

M_P=____________

e) What is the Kinetic Energy of the system immediately before M_B hits the ground?

KE=____________

Problem 14 : A merry-go-round of mass M=120kg and radius 5m rotates with an angular velocity ω_o=3rad/s. A block of mass M_A=90kg sits a distance R_A=4m from the axis of rotation. At time t=0s, Bob (m_B=70kg) jumps on the merry-go-round so that he lands a distance of R_B=3m from the axis of rotation. Assume the merry-go-round has a moment of inertia given by: $I=MR^2/2$.



a) What are the moments of inertia of the system before (just M_A and merry-go-round) and after (M_A, merry-go-round, and Bob) Bob jumps on?

I_o=_______________

I'=_______________

b) What is the angular velocity of the system after Bob jumps on?

ω'=_______________

c) If Bob wants the merry go round to return to its original angular velocity, where should he place the mass M_A, assuming he stays at R_B?

new R_A=_______________

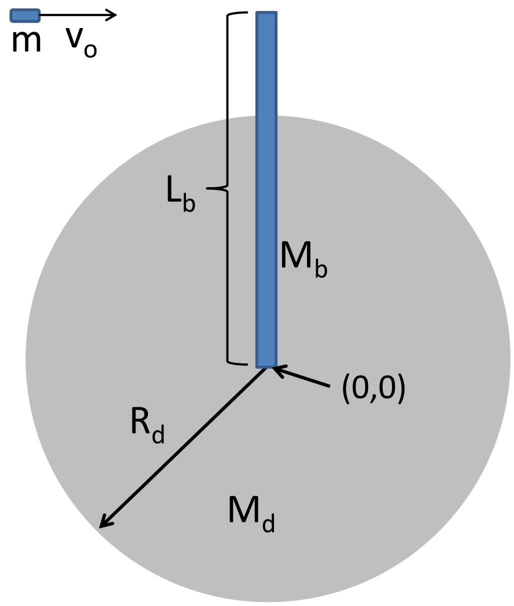
d) Determine the Kinetic energy of the system before Bob jumps on (KE_o), after Bob jumps on(KE'), and after Bob moves the mass in part c (KE'')

KE_o=_______________

KE'=_______________

KE''=_______________

Problem 15 : A system consists of circular disc (M_d=6kg, R_d=2m), onto which is affixed a rectangular block (M_b=4kg, L_b=3m), as shown in the figure below. The system starts at rest. the moment of inertia for the rectangle rotating about its center of mass is $I=ML^2/12$.



a) Where is the center mass of the system (disc + block) in the xy plane?

$\vec{r}_{cm}$ =________________

‘

b) What is the moment of inertia of the system (disc + block) *about the center of mass*? Hint: you will have to use the parallel axis theorem twice.

I=________________

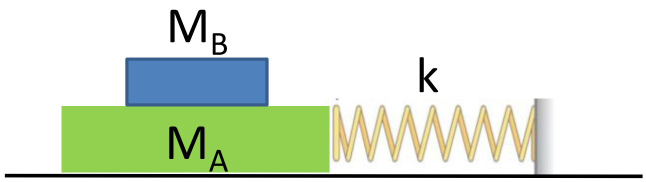
c) A smaller mass (m=100g) is shot into the very end of the of the rectangle with a velocity of 300m/s. The smaller mass embeds in the larger rectangle. What is the velocity of the center of mass of the system (now disc, rectangle, and bullet) immediately after the collision?

$$\vec{v}_{cm} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

d) With what angular frequency does the system rotate around the center of mass following the collision?

$$\omega_{cm} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Problem 16 : Mass A (25kg) is fixed to a spring of spring constant k=500N/m and slides on a frictionless surface. A second mass, Mass B (10kg), sits above Mass A. The coefficient of static friction between the surfaces of A and B is μ_s=0.7.



a) What is the natural frequency of oscillation of this system?

$\omega_o =$________________

b) What is the maximum Force which MA can exert on Mb without the surfaces slipping?

$F_{\max} =$________________

c) The system is started by giving the system an initial positive velocity at t=0 and x=0. Assuming the block stays on for that initial kick, what is the largest value of initial velocity the system can be given so that it oscillates without the blocks slipping?

v_{omax}=_______________

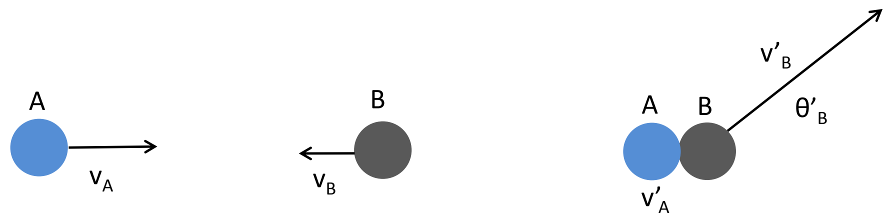
d) Give the equation of motion for the system started with the velocity in part (c)

x(t)=______________________

e) What is the total energy of this system?

E=______________________

Problem 17 : Two balls collide head on, with Ball A (M_A=2kg) traveling at a speed of 6m/s in the +x direction and Ball B (M_B=1kg) traveling with an initial velocity of -5m/s in the –x direction. After the collision, Mass B leaves the collision at an angle θ'_B=30° and a speed of v'_B=4m/s.



a) What is the final momentum of the system (both balls)?

$\vec{p}_f$ =________________

b) What is the final speed and angle for Ball A?

v'_A =________________

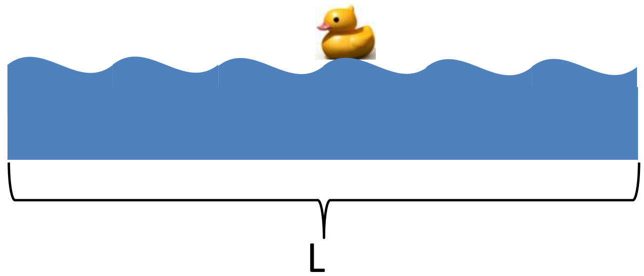
θ'_A =________________

c) Now assume the collision is elastic, but you do not know the final speed of Ball B? Determine what the final speeds of each ball must be.

v'_A =___I get 3.374m/s___θ_A'=-39.6°________

v'_B =___I get 8.615 m/s___________

Problem 18 : You are making (periodic) waves in the bathtub and you see Rubber Ducky bobbing up and down. You note that Ducky completes 7 cycles in 12 seconds.



a) What is the period and frequency of the waves you are generating?

ω =______________

T=______________

b) You do some more tests and you note that it takes 3 seconds for one of your waves to travel across the tub (L=1.5m). What is the wavelength and wavevector of the waves you are making?

λ =______________

k=______________

c) You do some more tests and you note that Ducky's ***peak to peak*** motion is 2cm. Give the expression for the waves you are generating.

$$D(x,t) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

d) Are the waves you are generating transverse or longitudinal?