PROBLEM 4.34

A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

<table>
<thead>
<tr>
<th></th>
<th>Aluminum</th>
<th>Brass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>70 GPa</td>
<td>105 GPa</td>
</tr>
<tr>
<td>Allowable stress</td>
<td>100 MPa</td>
<td>160 MPa</td>
</tr>
</tbody>
</table>

SOLUTION

Use aluminum as the reference material.

For aluminum, \( n = 1.0 \)

For brass, \( n = \frac{E_b}{E_a} = \frac{105}{70} = 1.5 \)

Values of \( n \) are shown on the sketch.

For the transformed section,

\[
I_1 = \frac{n_1 b h_1^3}{12} = \frac{1.5}{12}(8)^3 = 32.768 \times 10^3 \text{ mm}^4
\]

\[
I_2 = \frac{n_2 b_2}{12} \left( H_2^3 - h_2^3 \right) = \frac{1.0}{12}(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4
\]

\[
I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4
\]

\[
I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4
\]

\[
|\sigma| = \left| \frac{nM_y}{I} \right| \quad M = \left| \frac{\sigma I}{n y} \right|
\]

Aluminum:

\( n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa} \)

\[
M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}
\]

Brass:

\( n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa} \)

\[
M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}
\]

Choose the smaller value.

\( M = 887 \text{ N} \cdot \text{m} \)
**PROBLEM 4.41**

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is $1.8 \times 10^6$ psi and for steel, $29 \times 10^6$ psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip⋅in., determine the maximum stress in (a) the wood, (b) the steel.

**SOLUTION**

Use wood as the reference material.

For wood, $n = 1$

For steel, $n = E_s / E_w = 29 / 1.8 = 16.1111$

Transformed section: $\text{①} = \text{wood} \quad \text{②} = \text{steel}$

<table>
<thead>
<tr>
<th></th>
<th>$A$, in$^2$</th>
<th>$nA$, in$^2$</th>
<th>$\bar{y}_0$</th>
<th>$nA\bar{y}_0$, in$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>72</td>
<td>72</td>
<td>6</td>
<td>432</td>
</tr>
<tr>
<td>②</td>
<td>2.5</td>
<td>40.278</td>
<td>-0.25</td>
<td>-10.069</td>
</tr>
</tbody>
</table>

The neutral axis lies 3.758 in. above the wood-steel interface.

$$
I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1}{12}(6)(12)^3 + (72)(6 - 3.758)^2 = 1225.91 \text{ in}^4
$$

$$
I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{16.1111}{12}(5)(0.5)^3 + (40.278)(3.758 + 0.25)^2 = 647.87 \text{ in}^4
$$

$$
I = I_1 + I_2 = 1873.77 \text{ in}^4
$$

$$
M = 450 \text{ kip} \cdot \text{in.} \quad \sigma = -\frac{nMy}{I}
$$

(a) Wood: $n = 1$, $y = 12 - 3.758 = 8.242$ in.

$$
\sigma_w = -\frac{(1)(450)(8.242)}{1873.77} = -1.979 \text{ ksi} \quad \sigma_w = -1.979 \text{ ksi}
$$

(b) Steel: $n = 16.1111$, $y = 3.758 - 0.5 = -4.258$ in.

$$
\sigma_s = -\frac{(16.1111)(450)(-4.258)}{1873.77} = 16.48 \text{ ksi} \quad \sigma_s = 16.48 \text{ ksi}
$$
PROBLEM 4.42

The 6×12-in. timber beam has been strengthened by bolting to it the steel reinforcement shown. The modulus of elasticity for wood is $1.8 \times 10^6$ psi and for steel, $29 \times 10^6$ psi. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 450$ kip in., determine the maximum stress in (a) the wood, (b) the steel.

SOLUTION

Use wood as the reference material. For wood, $n = 1$

For steel, $n = \frac{E_s}{E_w} = \frac{29 \times 10^6}{1.8 \times 10^6} = 16.1111$

For C8 × 11.5 channel section, $A = 3.38 \text{ in}^2$, $t_w = 0.220 \text{ in.}$, $\bar{y} = 0.571 \text{ in.}$, $I_y = 1.32 \text{ in}^4$

For the composite section, the centroid of the channel (part 1) lies 0.571 in. above the bottom of the section. The centroid of the wood (part 2) lies $0.220 + 6.00 = 6.22 \text{ in.}$ above the bottom.

Transformed section:

<table>
<thead>
<tr>
<th>Part</th>
<th>$A$, in$^2$</th>
<th>$nA$, in$^2$</th>
<th>$\bar{y}$, in.</th>
<th>$nA\bar{y}$, in$^3$</th>
<th>$d$, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.38</td>
<td>54.456</td>
<td>0.571</td>
<td>31.091</td>
<td>3.216</td>
</tr>
<tr>
<td>2</td>
<td>72</td>
<td>72</td>
<td>6.22</td>
<td>447.84</td>
<td>2.433</td>
</tr>
<tr>
<td>Σ</td>
<td>126.456</td>
<td></td>
<td>478.93</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\bar{y}_0 = \frac{478.93 \text{ in}^3}{126.456 \text{ in}^2} = 3.787 \text{ in.}$  
$d = |\bar{y}_0 - \bar{y}|$

The neutral axis lies 3.787 in. above the bottom of the section.

$I_1 = n_1\bar{I}_1 + n_1A_1d_1^2 = (16.1111)(1.32) + (54.456)(3.216)^2 = 584.49 \text{ in}^4$

$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1}{12}(6)(12)^3 + (72)(2.433)^2 = 1290.20 \text{ in}^4$

$I = I_1 + I_2 = 1874.69 \text{ in}^4$

$M = 450 \text{ kip in.}$

$\sigma = -\frac{nMy}{I}$

(a) Wood: $n = 1$, $y = 12 + 0.220 - 3.787 = 8.433 \text{ in.}$

$\sigma_w = -\frac{(1)(450)(8.433)}{1874.69} = -2.02 \text{ ksi}$

(b) Steel: $n = 16.1111$, $y = -3.787 \text{ in.}$

$\sigma_s = -\frac{(16.1111)(450)(-3.787)}{1874.67} = 14.65 \text{ ksi}$

PROPRIETARY MATERIAL. Copyright © 2015 McGraw-Hill Education. This is proprietary material solely for authorized instructor use. Not authorized for sale or distribution in any manner. This document may not be copied, scanned, duplicated, forwarded, distributed, or posted on a website, in whole or part.

490
PROBLEM 4.51

Knowing that the bending moment in the reinforced concrete beam is $+100 \text{ kip} \cdot \text{ft}$ and that the modulus of elasticity is $3.625 \times 10^6 \text{ psi}$ for the concrete and $29 \times 10^6 \text{ psi}$ for the steel, determine (a) the stress in the steel, (b) the maximum stress in the concrete.

**SOLUTION**

\[ n = \frac{E_s}{E_c} = \frac{29 \times 10^6}{3.625 \times 10^6} = 8.0 \]

\[ A_s = (4) \left( \frac{\pi}{4} \right) (1)^2 = 3.1416 \text{ in}^2 \quad nA_s = 25.133 \text{ in}^2 \]

Locate the neutral axis.

\[ (24)(4)(x + 2) + (12x) \left( \frac{x}{2} \right) - (25.133)(17.5 - 4 - x) = 0 \]

\[ 96x + 192 + 6x^2 - 339.3 + 25.133x = 0 \quad \text{or} \quad 6x^2 + 121.133x - 147.3 = 0 \]

Solve for \( x \).

\[ x = \frac{-121.133 + \sqrt{(121.133)^2 + (4)(6)(147.3)}}{2(6)} = 1.150 \text{ in.} \]

\[ d_3 = 17.5 - 4 - x = 12.350 \text{ in.} \]

\[ I_1 = \frac{1}{12}bh_1^3 + A_s d_1^2 = \frac{1}{12} (24)(4)^3 + (24)(4)(3.150)^2 = 1080.6 \text{ in}^4 \]

\[ I_2 = \frac{1}{3}bh_2 x^3 = \frac{1}{3} (12)(1.150)^3 = 6.1 \text{ in}^4 \]

\[ I_3 = nA_s d_3^2 = (25.133)(12.350)^2 = 3833.3 \text{ in}^4 \]

\[ I = I_1 + I_2 + I_3 = 4920 \text{ in}^4 \]

\[ \sigma = -\frac{nM_y}{I} \quad \text{where} \quad M = 100 \text{ kip} \cdot \text{ft} = 1200 \text{ kip} \cdot \text{in.} \]

(a) Steel:

\[ n = 8.0 \quad y = -12.350 \text{ in.} \]

\[ \sigma_s = -\frac{(8.0)(1200)(-12.350)}{4920} = 24.1 \text{ ksi} \]

(b) Concrete:

\[ n = 1.0, \quad y = 4 + 1.150 = 5.150 \text{ in.} \]

\[ \sigma_c = -\frac{(1.0)(1200)(5.150)}{4920} = -1.256 \text{ ksi} \]