PROBLEM 5.44

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value \((a)\) of the shear, \((b)\) of the bending moment.

\[ \sum \Delta M_B = 0: \]
\[ -3A + (1)(4) + (0.5)(4) = 0 \]
\[ A = 2 \text{ kN} \uparrow \]
\[ \sum \Delta M_A = 0: 3B - (2)(4) - (2.5)(4) = 0 \]
\[ B = 6 \text{ kN} \uparrow \]

Shear diagram:
A to C: \(V = 2 \text{ kN}\)
C to D: \(V = 2 - 4 = -2 \text{ kN}\)
D to B: \(V = -2 - 4 = -6 \text{ kN}\)

Areas of shear diagram:
A to C: \[\int V \, dx = (1)(2) = 2 \text{ kN} \cdot \text{m}\]
C to D: \[\int V \, dx = (1)(-2) = -2 \text{ kN} \cdot \text{m}\]
D to E: \[\int V \, dx = (1)(-6) = -6 \text{ kN} \cdot \text{m}\]

Bending moments:
\[ M_A = 0 \]
\[ M_C^- = 0 + 2 = 2 \text{ kN} \cdot \text{m} \]
\[ M_C^+ = 2 + 4 = 6 \text{ kN} \cdot \text{m} \]
\[ M_D^- = 6 - 2 = 4 \text{ kN} \cdot \text{m} \]
\[ M_D^+ = 4 + 2 = 6 \text{ kN} \cdot \text{m} \]
\[ M_B = 6 - 6 = 0 \]

\((a)\) \[|V|_{\text{max}} = 6.00 \text{ kN} \]
\((b)\) \[|M|_{\text{max}} = 6.00 \text{ kN} \cdot \text{m} \]
PROBLEM 5.45

Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (a) of the shear, (b) of the bending moment.

SOLUTION

Shear:

\[ \sum M_A = 0: \]
\[ 0.075 F_{EF} - (0.2)(300) - (0.6)(300) = 0 \]
\[ F_{EF} = 3.2 \times 10^3 \text{ N} \]

\[ \sum F_x = 0: \]  \[ A_x - F_{EF} = 0 \]
\[ A_x = 3.2 \times 10^3 \text{ N} \]

\[ \sum F_y = 0: \]  \[ A_y - 300 - 300 = 0 \]
\[ A_y = 600 \text{ N} \]

Couple at D:  \[ M_D = (0.075)(3.2 \times 10^3) \]
\[ = 240 \text{ N} \cdot \text{m} \]

Shear:

A to C:  \[ V = 600 \text{ N} \]
C to B:  \[ V = 600 - 300 = 300 \text{ N} \]

Areas under shear diagram:

A to C:  \[ \int V \, dx = (0.2)(600) = 120 \text{ N} \cdot \text{m} \]
C to D:  \[ \int V \, dx = (0.2)(300) = 60 \text{ N} \cdot \text{m} \]
D to B:  \[ \int V \, dx = (0.2)(300) = 60 \text{ N} \cdot \text{m} \]

Bending moments:

\[ M_A = 0 \]
\[ M_C = 0 + 120 = 120 \text{ N} \cdot \text{m} \]
\[ M_D^- = 120 + 60 = 180 \text{ N} \cdot \text{m} \]
\[ M_D^+ = 180 - 240 = -60 \text{ N} \cdot \text{m} \]
\[ M_B = -60 + 60 = 0 \]

Maximum \[ |V| = 600 \text{ N} \]

Maximum \[ |M| = 180.0 \text{ N} \cdot \text{m} \]
PROBLEM 5.50

Determine \((a)\) the equations of the shear and bending-moment curves for the beam and loading shown, \((b)\) the maximum absolute value of the bending moment in the beam.

SOLUTION

\[
\frac{dV}{dx} = -w = -w_0\left(\frac{x}{L}\right)^{1/2} = -\frac{w_0 x^{1/2}}{L^{1/2}}
\]

\[
V = -\frac{2}{3} w_0 \frac{x^{3/2}}{L^{1/2}} + C_1
\]

\[
V = 0 \quad \text{at} \quad x = L
\]

\[
0 = -\frac{2}{3} w_0 L + C_1 \quad C_1 = \frac{2}{3} w_0 L
\]

\[
V = \frac{2}{3} w_0 L - \left(\frac{2}{3}\right) \frac{w_0 x^{3/2}}{L^{1/2}}
\]

\[
\frac{dM}{dx} = V \quad M = C_2 + \frac{2}{3} w_0 L x - \frac{2}{3} \frac{w_0 x^{5/2}}{L^{1/2}}
\]

\[
M = 0 \quad \text{at} \quad x = L \quad 0 = C + \frac{2}{3} w_0 L^2 - \frac{4}{15} w_0 L^2 \quad C_2 = -\frac{2}{5} w_0 L^2
\]

\[
M = \frac{2}{3} w_0 L x - \frac{4}{15} \frac{w_0 x^{5/2}}{L^{1/2}} - \frac{2}{5} w_0 L^2
\]

\[
|M|_{\text{max}} \quad \text{occurs at} \quad x = 0 \quad |M|_{\text{max}} = \frac{2}{5} w_0 L^2
\]
PROBLEM 5.58

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

\[ \Sigma M_B = 0: \quad R_A (16) - (4)(40)(25) - (24)(500) = 0 \quad R_A = 1000 \text{ lb} \quad \downarrow \]

\[ \Sigma M_A = 0: \quad R_B (16) - (20)(40)(25) - (40)(500) = 0 \quad R_B = 2500 \text{ lb} \quad \uparrow \]

Shear:

\[ V_A = -1000 \text{ lb} \]

\[ V_B = -1000 - (16)(25) = -1400 \text{ lb} \]

\[ V_{B+} = -1400 + 2500 = 1100 \text{ lb} \]

\[ V_C = 1100 - (24)(25) = 500 \text{ lb} \]

Areas of shear diagram:

(1) \( (1000 + 1400)(16) = 19,200 \text{ lb} \cdot \text{in.} \)

(2) \( (1100 + 500)(24) = 19,200 \text{ lb} \cdot \text{in.} \)

Bending moments:

\[ M_A = 0 \]

\[ M_B = 0 - 19,200 = -19,200 \text{ lb} \cdot \text{in.} \]

\[ M_C = -19,200 + 19,200 = 0 \]

Maximum \( |M| = 19.2 \text{ kip} \cdot \text{in.} \)

For S4 \( \times 7.7 \) rolled-steel section, \( S = 3.03 \text{ in}^3 \)

Normal stress:

\[ \sigma = \frac{|M|}{S} = \frac{19.2}{3.03} = 6.34 \text{ ksi} \]
PROBLEM 5.59

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

Reaction:

\[ \sum M_B = 0: \quad -4A + 60 + (80)(1.6)(2) - 12 = 0 \]
\[ A = 76 \text{kN} \uparrow \]

Shear:

\[ V_A = 76 \text{kN} \]
\[ A \text{ to } C: \quad V = 76.0 \text{kN} \downarrow \]
\[ V_D = 76 - (80)(1.6) = -52 \text{kN} \]
\[ D \text{ to } C: \quad V = -52 \text{kN} \]

Locate point where \( V = 0 \).
\[ V(x) = -80x + 76 = 0 \quad x = 0.95 \text{ m} \]

Areas of shear diagram:

\[ A \text{ to } C: \quad \int V \, dx = (1.2)(76) = 91.2 \text{ kN} \cdot \text{m} \]
\[ C \text{ to } E: \quad \int V \, dx = \frac{1}{2}(0.95)(76) = 36.1 \text{ kN} \cdot \text{m} \]
\[ E \text{ to } D: \quad \int V \, dx = \frac{1}{2}(0.65)(-52) = -16.9 \text{ kN} \cdot \text{m} \]
\[ D \text{ to } B: \quad \int V \, dx = (1.2)(-52) = -62.4 \text{ kN} \cdot \text{m} \]

Bending moments:

\[ M_A = -60 \text{ kN} \cdot \text{m} \]
\[ M_C = -60 + 91.2 = 31.2 \text{ kN} \cdot \text{m} \]
\[ M_E = 31.2 + 36.1 = 67.3 \text{ kN} \cdot \text{m} \]
\[ M_D = 67.3 - 16.9 = 50.4 \text{ kN} \cdot \text{m} \]
\[ M_B = 50.4 - 62.4 = -12 \text{ kN} \cdot \text{m} \]

For \( \text{W250} \times 80, \quad S = 983 \times 10^3 \text{mm}^3 \)

Normal stress:

\[ \sigma_{\text{max}} = \frac{|M|}{S} = \frac{67.3 \times 10^3 \text{ N} \cdot \text{m}}{983 \times 10^{-6} \text{m}^3} = 68.5 \times 10^6 \text{ Pa} \quad \sigma_m = 68.5 \text{ MPa} \downarrow \]