PROBLEM 5.67

For the beam and loading shown, design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of 1750 psi.

SOLUTION

Equivalent concentrated load:

\[ P = \left( \frac{1}{2} \right)(6)(1.2) = 3.6 \text{ kips} \]

Bending moment at A:

\[ M_A = (2)(3.6) = 7.2 \text{ kip} \cdot \text{ft} = 86.4 \text{ kip} \cdot \text{in.} \]

\[ S_{\text{min}} = \frac{|M_{\text{max}}|}{\sigma_{\text{all}}} = \frac{86.4}{1.75} = 49.37 \text{ in}^3 \]

For a square section,

\[ S = \frac{1}{6}a^3 \]

\[ a = \sqrt[3]{6S} \]

\[ a_{\text{min}} = \sqrt[3]{6(49.37)} \quad a_{\text{min}} = 6.67 \text{ in.} \]
PROBLEM 5.73

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

\[ \sigma_{\text{all}} = 160 \text{ MPa} \]

\[ S_{\text{min}} = \frac{M_{\text{max}}}{\sigma_{\text{all}}} = \frac{286 \text{ kN} \cdot \text{m}}{160 \text{ MPa}} = 1787 \times 10^{-6} \text{ m}^3 \]

\[ = 1787 \times 10^3 \text{ mm}^3 \]

<table>
<thead>
<tr>
<th>Shape</th>
<th>$S$ ($10^3 \text{ mm}^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W610 × 101</td>
<td>2520</td>
</tr>
<tr>
<td>W530 × 92</td>
<td>2080 ←</td>
</tr>
<tr>
<td>W460 × 113</td>
<td>2390</td>
</tr>
<tr>
<td>W410 × 114</td>
<td>2200</td>
</tr>
<tr>
<td>W360 × 122</td>
<td>2020</td>
</tr>
<tr>
<td>W310 × 143</td>
<td>2150</td>
</tr>
</tbody>
</table>

Use W530 × 92.
PROBLEM 5.74

Knowing that the allowable normal stress for the steel used is 160 MPa, select the most economical wide-flange beam to support the loading shown.

SOLUTION

\[ \sum M_D = 0: \quad -3.2B + (24)(3.2)(50) = 0 \quad B = 120 \text{ kN} \]
\[ \sum M_B = 0: \quad 3.2D - (0.8)(3.2)(50) = 0 \quad D = 40 \text{ kN} \]

Shear:
\[ V_A = 0 \]
\[ V_B = 0 - (0.8)(50) = -40 \text{ kN} \]
\[ V_B = -40 + 120 = 80 \text{ kN} \]
\[ V_C = 80 - (2.4)(50) = -40 \text{ kN} \]
\[ V_D = -40 + 0 = -40 \text{ kN} \]

Locate point E where \( V = 0 \).
\[ e = \frac{2.4 - e}{80} \quad 120e = 192 \]
\[ e = 1.6 \text{ m} \quad 2.4 - e = 0.8 \text{ m} \]

Areas:
\[ \text{A to B: } \int V \, dx = \left( \frac{1}{2} \right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m} \]
\[ \text{B to E: } \int V \, dx = \left( \frac{1}{2} \right)(1.6)(80) = 64 \text{ kN} \cdot \text{m} \]
\[ \text{E to C: } \int V \, dx = \left( \frac{1}{2} \right)(0.8)(-40) = -16 \text{ kN} \cdot \text{m} \]
\[ \text{C to D: } \int V \, dx = (0.8)(-40) = -32 \text{ kN} \cdot \text{m} \]

Bending moments:
\[ M_A = 0 \]
\[ M_B = 0 - 16 = -16 \text{ kN} \cdot \text{m} \]
\[ M_E = -16 + 64 = 48 \text{ kN} \cdot \text{m} \]
\[ M_C = 48 - 16 = 32 \text{ kN} \cdot \text{m} \]
\[ M_D = 32 - 32 = 0 \]
PROBLEM 5.74 (Continued)

Maximum $|M| = 48 \text{kN} \cdot \text{m} = 48 \times 10^3 \text{N} \cdot \text{m}$

$$\sigma_{\text{all}} = 160 \text{ MPa} = 160 \times 10^6 \text{ Pa}$$

$$S_{\text{min}} = \frac{|M|}{\sigma_{\text{all}}} = \frac{48 \times 10^3}{160 \times 10^6} = 300 \times 10^{-6} \text{ m}^3 = 300 \times 10^{-3} \text{ mm}^3$$

<table>
<thead>
<tr>
<th>Shape</th>
<th>$S(10^3 \text{mm}^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W310×32.7</td>
<td>415</td>
</tr>
<tr>
<td>W250×28.4</td>
<td>308 ←</td>
</tr>
<tr>
<td>W200×35.9</td>
<td>342</td>
</tr>
</tbody>
</table>

Lightest wide flange beam: W250×28.4 @ 28.4 kg/m
**PROBLEM 5.89**

Beams $AB$, $BC$, and $CD$ have the cross section shown and are pin-connected at $B$ and $C$. Knowing that the allowable normal stress is $+110$ MPa in tension and $-150$ MPa in compression, determine $(a)$ the largest permissible value of $w$ if beam $BC$ is not to be overstressed, $(b)$ the corresponding maximum distance $a$ for which the cantilever beams $AB$ and $CD$ are not overstressed.

**SOLUTION**

\[ M_B = M_C = 0 \]
\[ V_B = -V_C = \left( \frac{1}{2} \right) (7.2)w = 3.6w \]

Area $B$ to $E$ of shear diagram:

\[ \frac{1}{2} (3.6)(3.6w) = 6.48w \]

\[ M_E = 0 + 6.48w = 6.48w \]

Centroid and moment of inertia:

<table>
<thead>
<tr>
<th>Part</th>
<th>$A$ (mm$^2$)</th>
<th>$\bar{y}$ (mm)</th>
<th>$A\bar{y}$ (mm$^3$)</th>
<th>$d$ (mm)</th>
<th>$Ad^2$ (mm$^4$)</th>
<th>$\bar{I}$ (mm$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>2500</td>
<td>156.25</td>
<td>390,625</td>
<td>34.82</td>
<td>$3.031 \times 10^6$</td>
<td>$0.0326 \times 10^6$</td>
</tr>
<tr>
<td>②</td>
<td>1875</td>
<td>75</td>
<td>140,625</td>
<td>46.43</td>
<td>$4.042 \times 10^6$</td>
<td>$3.516 \times 10^6$</td>
</tr>
<tr>
<td>Σ</td>
<td>4375</td>
<td></td>
<td>531,250</td>
<td></td>
<td>$7.073 \times 10^6$</td>
<td>$3.548 \times 10^6$</td>
</tr>
</tbody>
</table>

\[ \bar{y} = \frac{531,250}{4375} = 121.43 \text{ mm} \]

\[ I = \Sigma Ad^2 + \Sigma \bar{I} = 10.621 \times 10^6 \text{ mm}^4 \]

<table>
<thead>
<tr>
<th>Location</th>
<th>$y$(mm)</th>
<th>$I/y(10^3 \text{ mm}^3)$</th>
<th>( \leftarrow ) also ( (10^{-6} \text{ m}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top</td>
<td>41.07</td>
<td>258.6</td>
<td></td>
</tr>
<tr>
<td>Bottom</td>
<td>$-121.43$</td>
<td>$-87.47$</td>
<td></td>
</tr>
</tbody>
</table>
PROBLEM 5.89 (Continued)

Bending moment limits: \( M = -\sigma I / y \)

Tension at \( E \): \(-((110 \times 10^6)(-87.47 \times 10^{-6})) = 9.622 \times 10^3 \text{ N} \cdot \text{m}\)

Compression at \( E \): \(-((-150 \times 10^{-6})(258.6 \times 10^{-6})) = 38.8 \times 10^3 \text{ N} \cdot \text{m}\)

Tension at \( A \) and \( D \): \(-((110 \times 10^6)(258.6 \times 10^{-6})) = -28.45 \times 10^3 \text{ N} \cdot \text{m}\)

Compression at \( A \) and \( D \): \(-((-150 \times 10^6)(-87.47 \times 10^{-6})) = -13.121 \times 10^3 \text{ N} \cdot \text{m}\)

(a) Allowable load \( w \): \(6.48w = 9.622 \times 10^3 \quad w = 1.485 \times 10^3 \text{ N/m} \quad w = 1.485 \text{ kN/m} \)

Shear at \( A \): \(V_A = (a + 3.6)w\)

Area \( A \) to \( B \) of shear diagram: \(\frac{1}{2}a(V_A + V_B) = \frac{1}{2}a(a + 7.2)w\)

Bending moment at \( A \) (also \( D \)): \(M_A = -\frac{1}{2}a(a + 7.2)w\)

\[-\frac{1}{2}a(a + 7.2)(4.485 \times 10^3) = -13.121 \times 10^3\]

(b) Distance \( a \):

\[\frac{1}{2}a^2 + 3.6a - 8.837 = 0\]

\[a = 1.935 \text{ m} \]

---

PROPRIETARY MATERIAL. Copyright © 2015 McGraw-Hill Education. This is proprietary material solely for authorized instructor use. Not authorized for sale or distribution in any manner. This document may not be copied, scanned, duplicated, forwarded, distributed, or posted on a website, in whole or part.