Effective Stress Concept

No Seepage

Total Stress ($\sigma$) at Point A

$$\sigma = H \gamma_w + (H_A - H) \gamma_{sat}$$

Where:

$\gamma_w = \text{Unit Weight of Water}$

$\gamma_{sat} = \text{Saturated Unit Weight of Soil}$

$H = \text{Height of water above Soil}$

$H_A = \text{Depth of Point A below water table}$

Saturated Soil Column (Figure 6.1. Das FGE (2005))
**Effective Stress Concept**

**No Seepage**

Total Stress ($\sigma$) can be divided into 2 Parts:

1. Portion carried by water in void spaces. **THIS IS THE PORE PRESSURE ($u$).**

2. Portion carried by soil solids at points of contact. **THIS IS THE EFFECTIVE STRESS ($\sigma'$).**
Effective Stress Concept

No Seepage

Saturated Soil Column (Figure 6.1. Das FGE (2005))

 Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line a-a)

Figure 6.1. Das FGE (2005)

Effective Stress ($\sigma'$) along Line a-a

$$\sigma' = \frac{P_1(v) + P_2(v) + P_3(v) + \ldots + P_n(v)}{A}$$
Effective Stress Concept

No Seepage

Effective Stress ($\sigma'$)
along Line $a-a$

$$\sigma' = \frac{P_1(v) + P_2(v) + P_3(v) + \ldots + P_n(v)}{\bar{A}}$$

Where:

- $P_1(v) = \text{Vertical Component of } P_1$
- $\bar{A} = \text{Cross-sectional Area of Soil Mass Under Consideration}$

Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line $a-a$)

Figure 6.1. Das FGE (2005)
Effective Stress Concept

No Seepage

Total Stress ($\sigma$) along Line a-a

$$\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{A} = \sigma' + u(1 - a'_s)$$

Where:

- $a_s = \text{Cross-section Area of Soil Contacts} = a_1 + a_2 + a_3 + \ldots + a_n$
- $\bar{A} = \text{Cross-sectional Area of Soil Mass Under Consideration}$
- $a'_s = a_s/\bar{A} = \text{Fraction of unit cross-sectional area of soil mass occupied by solid to solid contacts.}$

Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line a-a)

Figure 6.1. Das FGE (2005).
**Effective Stress Concept**

**No Seepage**

Total Stress ($\sigma$) along Line $a-a$

$$\sigma = \sigma' + u\left(\frac{A - a_s}{A}\right) = \sigma' + u(1 - a'_s)$$

$a'_s \approx 0$ (i.e. very small), so therefore:

$$= \sigma' + u \quad \text{or} \quad \sigma' = \sigma - u$$

**THE EFFECTIVE STRESS EQUATION**

*Figure 6.1. Das FGE (2005)*
**Effective Stress Concept**

**No Seepage**

**The Effective Stress Equation**

\[ \sigma' = \sigma - u \]

\[ \sigma' = \left[ H \gamma_w + (H_A - H) \gamma_{sat} \right] - H_A \gamma_w \]

- **\( \sigma \)** = Total Stress
- **\( u \)** = Pore Pressure
- **\( \gamma \)** = Submerged unit weight of soil
- **\( \gamma_w \)** = Water unit weight
- **\( \gamma_{sat} \)** = Saturated unit weight
- **\( H_A \)** = Height of soil column
- **\( A \)** = Cross-sectional area

*Figure 6.1. Das FGE (2005)*
Effective Stress Concept

No Seepage: Example Problem

Given Soil Profile (NTS):

- **CL**
  - \( \gamma = 102 \text{ lb/ft}^3 \)
  - \( \gamma_{sat} = 105 \text{ lb/ft}^3 \)

- **SM**
  - \( \gamma_{sat} = 115 \text{ lb/ft}^3 \)

Find:

- Total and Effective Stresses at Pts. A, B, C, & D.
14.330 SOIL MECHANICS
Effective Stress

**Effective Stress Concept**

NO SEEPAGE: EXAMPLE PROBLEM

**Given Soil Profile (NTS):**

- **CL**
  - \( \gamma = 102 \text{ lb/ft}^3 \)
  - \( \gamma_{sat} = 105 \text{ lb/ft}^3 \)

- **SM**
  - \( \gamma_{sat} = 115 \text{ lb/ft}^3 \)

**Find:**

Total and Effective Stresses at Pts. A, B, C, & D.

**@ Point A:**

\[
\sigma_A = \gamma_{CL} \times Z_A = 102 \frac{lb}{ft^3} (5 \text{ ft})
\]

\[
\sigma_A = 510 \frac{lb}{ft^2}
\]

\[
\sigma'_A = \sigma_A - u_A
\]

\( u_A = 0 \)

\[
\therefore \sigma'_A = \sigma_A = 510 \frac{lb}{ft^2}
\]
Effective Stress Concept

No Seepage: Example Problem

GIVEN SOIL PROFILE (NTS):

CL
\( \gamma = 102 \text{ lb/ft}^3 \)
\( \gamma_{\text{sat}} = 105 \text{ lb/ft}^3 \)

SM
\( \gamma_{\text{sat}} = 115 \text{ lb/ft}^3 \)

@ Point B:

\[ \sigma_B = \sigma_A + (\gamma_{\text{sat}} \cdot CL \times 4 \text{ ft}) \]

\[ \sigma_B = 510 \frac{\text{lb}}{\text{ft}^3} + 105 \frac{\text{lb}}{\text{ft}^3} (4 \text{ ft}) \]

\[ \sigma_B = 930 \frac{\text{lb}}{\text{ft}^2} \]

\[ \sigma'_B = \sigma_B - u_B \]

\[ u_B = \gamma_w \times 4 \text{ ft} = 62.4 \frac{\text{lb}}{\text{ft}^3} \times 4 \text{ ft} = 250 \frac{\text{lb}}{\text{ft}^2} \]

\[ \sigma'_B = \sigma_B - u_B = 930 \frac{\text{lb}}{\text{ft}^2} - 250 \frac{\text{lb}}{\text{ft}^2} \]

\[ \sigma'_B = 680 \frac{\text{lb}}{\text{ft}^2} \]
14.330 SOIL MECHANICS
Effective Stress

**Effective Stress Concept**

**No Seepage: Example Problem**

**GIVEN SOIL PROFILE (NTS):**

- **CL**
  - $\gamma = 102 \text{ lb/ft}^3$
  - $\gamma_{sat} = 105 \text{ lb/ft}^3$

- **SM**
  - $\gamma_{sat} = 115 \text{ lb/ft}^3$

**@ Point C:**

\[ \sigma_C = \sigma_B + (\gamma_{sat,SM} \times 6 \text{ ft}) \]
\[ \sigma_C = \frac{930}{\text{lb/ft}^3} + \frac{115}{\text{lb/ft}^3} \times 6 \text{ ft} \]
\[ \sigma_C = 1620 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma'_C = \sigma_C - u_C \]
\[ u_C = \gamma_w \times 10 \text{ ft} = 62.4 \frac{\text{lb}}{\text{ft}^3} \times 10 \text{ ft} = 624 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma'_C = \sigma_C - u_C = 1620 \frac{\text{lb}}{\text{ft}^2} - 624 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma'_C = 996 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma'_C = 1000 \frac{\text{lb}}{\text{ft}^2} \text{ (round to nearest 5 psf)} \]
Effective Stress Concept

No Seepage: Example Problem

@ Point D:

\[ \sigma_D = \sigma_B + (\gamma_{\text{sat,SM}} \times 12 \text{ ft}) \]
\[ \sigma_D = 930 \frac{\text{lb}}{\text{ft}^3} + 115 \frac{\text{lb}}{\text{ft}^3} (12 \text{ ft}) \]
\[ \sigma_D = 2310 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma_D' = \sigma_D - u_D \]
\[ u_D = \gamma_w \times 16 \text{ ft} = 62.4 \frac{\text{lb}}{\text{ft}^3} \times 16 \text{ ft} = 998 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma_D' = \sigma_D - u_D = 2310 \frac{\text{lb}}{\text{ft}^2} - 998 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma_D' = 1312 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma_D' = 1310 \frac{\text{lb}}{\text{ft}^2} \] (round to nearest 5 psf)
Effective Stress Concept

No Seepage: Example Problem

Total Stress ($\sigma$)

Depth from Existing Ground Surface (ft)

Pore Pressure ($u$)

Effective Stress ($\sigma'$)

Depth from Existing Ground Surface (ft)

CL

$\gamma = 102$ pcf
$\gamma_{sat} = 105$ pcf

SM

$\gamma_{sat} = 115$ pcf

$\gamma = 102$ pcf

Use 1310

Use 1000
Stresses @ Point A:
\[ \sigma_A = H_1 \gamma_w \]
\[ u_A = H_1 \gamma_w \]
\[ \sigma'_A = \sigma_A - u_A = 0 \]

Stresses @ Point B:
\[ \sigma_B = H_1 \gamma_w + H_2 \gamma_{sat} \]
\[ u_B = (H_1 + H_2 + h) \gamma_w \]
\[ \sigma'_B = \sigma_B - u_B \]
\[ \sigma'_B = (H_1 \gamma_w + H_2 \gamma_{sat}) - (H_1 + H_2 + h) \gamma_w \]
\[ \sigma'_B = H_2 (\gamma_{sat} - \gamma_w) - h \gamma_w \]
\[ \sigma'_B = H_2 \gamma' - h \gamma_w \]
Effective Stress Concept

Upward Seepage

Stresses @ Point C:

\[ \sigma_C = H_1 \gamma_w + z \gamma_{sat} \]
\[ u_C = (H_1 + z + \frac{h}{H_2} z) \gamma_w \]
\[ \sigma'_C = \sigma_C - u_C \]
\[ \sigma'_C = z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2} z \gamma_w \]
\[ \sigma'_C = z \gamma' - \frac{h}{H_2} z \gamma_w \]

NOTE: \[ i = \frac{h}{H_2} = \frac{\text{Change in Head}}{\text{Length of Water Flow}} \]

\[ \therefore \sigma'_C = z \gamma' - iz \gamma_w \]
**Effective Stress Concept**

**Upward Seepage**

Critical hydraulic gradient ($i_{cr}$)

$$\sigma'_C = z \gamma' - i_{cr} z \gamma_w = 0$$

No effective stress!

Known as Boiling or Quick Condition

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

For Most Soils:

$i_{cr}$ ranges from 0.9 to 1.1, with an average of 1

---

*Figure 6.3a. Das FGE (2005).*
Effective Stress Concept

Upward Seepage

Total stress, $\sigma$

Pore water pressure, $u$

Effective stress, $\sigma'$

$H_1\gamma_w$

$H_1\gamma_w + z\gamma_{sat}$

$(H_1 + z + iz)\gamma_w$

$z\gamma' - iz\gamma_w$

$H_1\gamma_w$

$H_2\gamma_{sat}$

$(H_1 + H_2 + h)\gamma_w$

$H_2\gamma' - h\gamma_w$

Figure 6.3b. Das FGE (2005).
Effective Stress Concept

Downward Seepage

**Stresses @ Point A:**
\[ \sigma_A = H_1 \gamma_w \]
\[ \sigma'_A = \sigma_A - u_A = 0 \]
\[ u_A = H_1 \gamma_w \]

**Stresses @ Point B:**
\[ \sigma_B = H_1 \gamma_w + H_2 \gamma_{sat} \]
\[ u_B = (H_1 + H_2 - h) \gamma_w \]
\[ \sigma'_B = \sigma_B - u_B \]
\[ \sigma'_B = (H_1 \gamma_w + H_2 \gamma_{sat}) - (H_1 + H_2 - h) \gamma_w \]
\[ \sigma'_B = H_2 (\gamma_{sat} - \gamma_w) + h \gamma_w \]
\[ \sigma'_B = H_2 \gamma' + h \gamma_w \]

*Figure 6.4a. Das FGE (2005).*
Effective Stress Concept

Downward Seepage

**Stresses @ Point C:**

\[ \sigma'_C = \sigma_C - u_C \]

\[ \sigma'_C = H_1 \gamma_w + z \gamma_{sat} - \left( H_1 + z - \frac{h}{H_2} z \right) \gamma_w \]

\[ \sigma'_C = z \gamma' + \frac{h}{H_2} z \gamma_w \]

**NOTE:** \( i = \frac{h}{H_2} = \frac{\text{Change in Head}}{\text{Length of Water Flow}} \)

\[ \therefore \sigma'_C = z \gamma' + i z \gamma_w \]

**Figure 6.4a.** Das FGE (2005).
Effective Stress Concept

Downward Seepage

Figure 6.4b. Das FGE (2005).
Effective Stress Concept

Partially Saturated Soil

\[ \sigma' = \sigma - u_a + \chi (u_a - u_w) \]

Where:
- \( u_a \) = Pore Air Pressure
- \( u_w \) = Pore Water Pressure
- \( \chi \) = Fraction of unit cross-sectional area of soil occupied by water.
  - \( \chi = 0 \) for dry soil; 1 for saturated soil.
- \( \chi \) depends on degree of saturation (S).
  - Also influenced by soil structure.

Figure 6.6. Das FGE (2005).
**Capillary Rise in Soils**

**Summing Forces in Vertical Direction**

\[
\left( \frac{\pi}{4}d^2 \right)h_c \gamma_w = \pi d T \cos \alpha
\]

\[
h_c = \frac{4T \cos \alpha}{d \gamma_w}
\]

**Where:**
- \( T \) = Surface Tension
- \( \alpha \) = Angle of Contact
- \( d \) = Capillary Tube Diameter

\( T, \alpha, \gamma_w \) remain constant

\[
h_c \propto \frac{1}{d}
\]

---

**Figure 8.19.** Principles of Geotechnical Engineering, Das (2006).
Capillary Rise in Soils

\[ h_c \propto \frac{1}{d} \]

**Figure 8.20.** Principles of Geotechnical Engineering, Das (2006).
14.330 SOIL MECHANICS
Effective Stress

**CAPILLARY RISE IN SOILS**

Hazen (1930)

\[ h_1 = \frac{C}{eD_{10}} \]

Where:
- \( D_{10} \) = Effective Size (mm)
- \( e \) = Void Ratio
- \( C \) = Constant (ranging from 10 mm\(^2\) to 50 mm\(^2\))

(a) Sandy soil  (b) Water
**Capillary Rise in Soils**

Table 8.2 (Das, PGE 2006). Approximate Range of Capillary Rise in Soils.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Range of Capillary Rise</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m</td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>0.1 – 0.2</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.3 – 1.2</td>
</tr>
<tr>
<td>Silt</td>
<td>0.75 – 7.5</td>
</tr>
<tr>
<td>Clay</td>
<td>7.5 - 23</td>
</tr>
</tbody>
</table>

**Effective Stress in Capillary Zone**

\[ \sigma' = \sigma - u \]

Saturated: \[ u = -h \gamma w \]

Partially Saturated: \[ u = -h \left( \frac{S}{100} \right) \gamma w \]
**Effective Stress**

**WITH NO SEEPAGE**
(i.e. STATIC CONDITIONS)

\[
\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)
\]

\[
(\gamma_{sat} - \gamma_w) = \gamma'
\]

Height of soil column = \(z\)

\(\gamma' = \) Submerged unit weight of soil

**Effective Stress:**

\[\sigma' = z\gamma'\]

**Effective Force:**

\[P_1' = z\gamma' A\]

Where:

\(A = \) Area

---

**Figure 6.1.** Das FGE (2005).
SEEPAGE FORCE: UPWARD SEEPAGE

**Effective Stress**

\[ \sigma' = z \gamma' \]

**Effective Force**

\[ P'_1 = z \gamma' A \]

**No Seepage:**

\[ \sigma' = z \gamma' \quad P'_1 = z \gamma' A \]

**With Seepage:**

\[ \sigma' = z \gamma' - iz \gamma_w \quad P'_2 = (z \gamma' - iz \gamma_w) A \]

**Decrease of Total Force Due to Seepage:**

\[ P'_1 - P'_2 = iz \gamma_w A \]

**Seepage Force per Unit Volume:**

\[ \frac{P'_1 - P'_2}{(\text{Soil Volume})} = iz \gamma_w A = i \gamma_w \]
SEEPAGE FORCE SUMMARY

NO SEEPAGE

Volume of soil = zA

\[(\gamma' - i\gamma_w)A\] = \[\gamma'A\] + \[i\gamma_wA = \text{seepage force}\]

UPWARD SEEPAGE

Volume of soil = zA

\[\gamma'A\]

DOWNWARD SEEPAGE

Volume of soil = zA

\[(\gamma' + i\gamma_w)A\] = \[\gamma'A\] + \[i\gamma_wA = \text{seepage force}\]