EFFECTIVE STRESS CONCEPT

NO SEEPAGE

Total Stress ($\sigma$) at Point A

\[ \sigma = H\gamma_w + (H_A - H)\gamma_{sat} \]

Where:

- $\gamma_w$ = Unit Weight of Water
- $\gamma_{sat}$ = Saturated Unit Weight of Soil
- $H$ = Height of water above Soil
- $H_A$ = Depth of Point A below water table
EFFECTIVE STRESS CONCEPT

NO SEEPAGE

Total Stress ($\sigma$) can be divided into 2 Parts:

1. Portion carried by water in void spaces. **THIS IS THE PORE PRESSURE ($u$).**

2. Portion carried by soil solids at points of contact. **THIS IS THE EFFECTIVE STRESS ($\sigma'$).**
Effective Stress Concept

Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line a-a)

Effective Stress ($\sigma'$) along Line a-a

$$\sigma' = \frac{P_1(v) + P_2(v) + P_3(v) + \ldots + P_n(v)}{A}$$
Effective Stress Concept

**Effective Stress (σ’)** along Line a-a

\[
\sigma' = \frac{P_1(v) + P_2(v) + P_3(v) + \ldots + P_n(v)}{\bar{A}}
\]

Where:

- \( P_1(v) = \) Vertical Component of \( P_1 \)
- \( \bar{A} = \) Cross-sectional Area of Soil Mass Under Consideration

Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line a-a)

*Figure 6.1. Das FGE (2005)*
Effect of Seepage
Forces acting at Soil Particle Points of Contact at level of Point A (i.e. along Line a-a) 

Figure 6.1. Das FGE (2005).

EFFECTIVE STRESS CONCEPT

NO SEEPAGE

Total Stress ($\sigma$) along Line $a-a$

$$\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{\bar{A}} = \sigma' + u(1 - a'_s)$$

Where:

- $a_s$ = Cross-section Area of Soil Contacts = $a_1 + a_2 + a_3 + \ldots + a_n$
- $\bar{A}$ = Cross-sectional Area of Soil Mass Under Consideration
- $a'_s = \frac{a_s}{\bar{A}}$ = Fraction of unit cross-sectional area of soil mass occupied by solid to solid contacts.
EFFECTIVE STRESS CONCEPT

NO SEEPAGE

Figure 6.1. Das FGE (2005).

Total Stress (σ) along Line a-a

\[ \sigma = \sigma' + \frac{u(A - a_s)}{A} = \sigma' + u(1 - a'_s) \]

\( a'_s \approx 0 \) (i.e. very small), so therefore:

\[ \sigma = \sigma' + u \quad \text{or} \quad \sigma' = \sigma - u \]

THE EFFECTIVE STRESS EQUATION
EFFECTIVE STRESS CONCEPT

NO SEEPAGE

THE EFFECTIVE STRESS EQUATION

\[ \sigma' = \sigma - u \]

\[ \sigma' = \left[ H\gamma_w + (H_A - H)\gamma_{sat} \right] - H_A\gamma_w \]

- \( \sigma = \text{total stress} \)
- \( u = \text{water pressure} \)

\[ \sigma' = \left( H_A - H \right) \left( \gamma_{sat} - \gamma_w \right) \]

\( \gamma' = \text{Submerged unit weight of soil} \)

\( \gamma_{sat} - \gamma_w = \gamma' \)

**Figure 6.1.** Das FGE (2005).
EFFECTIVE STRESS CONCEPT

NO SEEPAGE - EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):

- **CL**
  - $\gamma = 102$ lb/ft$^3$
  - $\gamma_{sat} = 105$ lb/ft$^3$

- **SM**
  - $\gamma_{sat} = 115$ lb/ft$^3$

FIND:

Total and Effective Stresses at Pts. A, B, C, & D.
GIVEN SOIL PROFILE (NTS):

CL
\[ \gamma = 102 \text{ lb/ft}^3 \]
\[ \gamma_{\text{sat}} = 105 \text{ lb/ft}^3 \]

SM
\[ \gamma_{\text{sat}} = 115 \text{ lb/ft}^3 \]

FIND:
Total and Effective Stresses at Pts. A, B, C, & D.

@ Point A:
\[ \sigma_A = \gamma_{\text{CL}} \times Z_A = 102 \frac{\text{lb}}{\text{ft}^3} (5 \text{ ft}) \]
\[ \sigma_A = 510 \frac{\text{lb}}{\text{ft}^2} \]
\[ \sigma'_A = \sigma_A - u_A \]
\[ u_A = 0 \]
\[ \therefore \sigma'_A = \sigma_A = 510 \frac{\text{lb}}{\text{ft}^2} \]
**EFFECTIVE STRESS CONCEPT**

NO SEEPAGE - EXAMPLE PROBLEM

**GIVEN SOIL PROFILE (NTS):**

**CL**
- \( \gamma = 102 \text{ lb/ft}^3 \)
- \( \gamma_{sat} = 105 \text{ lb/ft}^3 \)

**SM**
- \( \gamma_{sat} = 115 \text{ lb/ft}^3 \)

@ Point B:

\[
\sigma_B = \sigma_A + (\gamma_{sat,CL} \times 4 \text{ ft})
\]

\[
\sigma_B = 510 \frac{lb}{ft^3} + 105 \frac{lb}{ft^3} \times 4 \text{ ft}
\]

\[
\sigma_B = 930 \frac{lb}{ft^2}
\]

\[
\sigma'_B = \sigma_B - u_B
\]

\[
u_B = \gamma_w \times 4 \text{ ft} = 62.4 \frac{lb}{ft^3} \times 4 \text{ ft} = 250 \frac{lb}{ft^2}
\]

\[
\sigma'_B = \sigma_B - u_B = 930 \frac{lb}{ft^2} - 250 \frac{lb}{ft^2}
\]

\[
\sigma'_B = 680 \frac{lb}{ft^2}
\]
EFFECTIVE STRESS CONCEPT
NO SEEPAGE - EXAMPLE PROBLEM

GIVEN SOIL PROFILE (NTS):

\[ \begin{align*}
\gamma_{\text{CL}} &= 102 \text{ lb/ft}^3 \\
\gamma_{\text{sat, CL}} &= 105 \text{ lb/ft}^3 \\
\gamma_{\text{SM}} &= 115 \text{ lb/ft}^3 \\
\gamma_{\text{sat, SM}} &= 115 \text{ lb/ft}^3
\end{align*} \]

\[ \begin{align*}
A &\quad 5\text{ft} & B &\quad 9\text{ft} \\
C &\quad 6\text{ft} & D &\quad 12\text{ft}
\end{align*} \]

\[ \begin{align*}
\sigma_C &= \sigma_B + (\gamma_{\text{sat, SM}} \times 6\text{ ft}) \\
\sigma_C &= 930 \frac{\text{lb}}{\text{ft}^3} + 115 \frac{\text{lb}}{\text{ft}^3} (6 \text{ ft}) \\
\sigma_C &= 1620 \frac{\text{lb}}{\text{ft}^2} \\
\sigma_C' &= \sigma_C - u_C \\
u_C &= \gamma_w \times 10\text{ ft} = 62.4 \frac{\text{lb}}{\text{ft}^3} \times 10 \text{ ft} = 624 \frac{\text{lb}}{\text{ft}^2} \\
\sigma_C' &= 1620 \frac{\text{lb}}{\text{ft}^2} - 624 \frac{\text{lb}}{\text{ft}^2} \\
\sigma_C' &= 996 \frac{\text{lb}}{\text{ft}^2} \\
\sigma_C' &= 1000 \frac{\text{lb}}{\text{ft}^2} \quad \text{(round to nearest 5 psf)}
\end{align*} \]

@ Point C:
**EFFECTIVE STRESS CONCEPT**

**NO SEEPAGE - EXAMPLE PROBLEM**

**GIVEN SOIL PROFILE (NTS):**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Density $\gamma$</th>
<th>Saturation Density $\gamma_{sat}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CL</td>
<td>102 lb/ft$^3$</td>
<td>105 lb/ft$^3$</td>
</tr>
<tr>
<td>SM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**@ Point D:**

\[
\sigma_D = \sigma_B + (\gamma_{sat,SM} \times 12 \text{ ft})
\]

\[
\sigma_D = 930 \frac{lb}{ft^3} + 115 \frac{lb}{ft^3} (12 \text{ ft})
\]

\[
\sigma_D = 2310 \frac{lb}{ft^2}
\]

\[
\sigma'_D = \sigma_D - u_D
\]

\[
\begin{align*}
    u_D &= \gamma_w \times 16 \text{ ft} = 62.4 \frac{lb}{ft^3} \times 16 \text{ ft} = 998 \frac{lb}{ft^2} \\
    \sigma'_D &= \sigma_D - u_D = 2310 \frac{lb}{ft^2} - 998 \frac{lb}{ft^2} \\
    \sigma'_D &= 1312 \frac{lb}{ft^2}
\end{align*}
\]

\[
\sigma'_D = 1310 \frac{lb}{ft^2} \quad \text{(round to nearest 5 psf)}
\]
EFFECTIVE STRESS CONCEPT

NO SEEPAGE - EXAMPLE PROBLEM

TOTAL STRESS \((\sigma)\)  
Pore Pressure \((u)\)  
Effective Stress \((\sigma')\)

- **CL**  
  \(\gamma = 102\) pcf  
  \(\gamma_{sat} = 105\) pcf

- **SM**  
  \(\gamma = 115\) pcf

Use 1000  
Use 1310
**EFFECTIVE STRESS CONCEPT**

**UPWARD SEEPAGE**

**Stresses @ Point A:**

\[ \sigma_A = H_1 \gamma_w \]
\[ u_A = H_1 \gamma_w \]
\[ \sigma'_A = \sigma_A - u_A = 0 \]

**Stresses @ Point B:**

\[ \sigma_B = H_1 \gamma_w + H_2 \gamma_{sat} \]
\[ u_B = (H_1 + H_2 + h) \gamma_w \]
\[ \sigma'_B = \sigma_B - u_B \]
\[ \sigma'_B = (H_1 \gamma_w + H_2 \gamma_{sat}) - (H_1 + H_2 + h) \gamma_w \]
\[ \sigma'_B = H_2 (\gamma_{sat} - \gamma_w) - h \gamma_w \]

*Figure 6.3a. Das FGE (2005).*
Effective Stress Concept

Stresses at Point C:

\[ \sigma_C = H_1 \gamma_w + z \gamma_{sat} \]
\[ u_C = (H_1 + z + \frac{h}{H_2} z) \gamma_w \]
\[ \sigma'_C = \sigma_C - u_C \]
\[ \sigma'_C = z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2} z \gamma_w \]
\[ \sigma'_C = z \gamma' - \frac{h}{H_2} z \gamma_w \]

**NOTE:** \[ i = \frac{h}{H_2} = \frac{\text{Change in Head}}{\text{Length of Water Flow}} \]

\[ \therefore \sigma'_C = z \gamma' - iz \gamma_w \]
EFFECTIVE STRESS CONCEPT
UPWARD SEEPAGE

CRITICAL HYDRAULIC GRADIENT ($i_{cr}$)

$$\sigma'_C = z\gamma' - i_{cr} z\gamma_w = 0$$

NO EFFECTIVE STRESS!
Known as Boiling or Quick Condition

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

For Most Soils:
$i_{cr}$ ranges from 0.9 to 1.1, with an average of 1
EFFECTIVE STRESS CONCEPT

UPWARD SEEPAGE

Figure 6.3b. Das FGE (2005).
**Effective Stress Concept**

**Downward Seepage**

**Stresses @ Point A:**

\[ \sigma_A = H_1 \gamma_w \]

\[ u_A = H_1 \gamma_w \]

\[ \sigma'_A = \sigma_A - u_A = 0 \]

**Stresses @ Point B:**

\[ \sigma_B = H_1 \gamma_w + H_2 \gamma_{sat} \]

\[ u_B = (H_1 + H_2 - h) \gamma_w \]

\[ \sigma'_B = \sigma_B - u_B \]

\[ \sigma'_B = (H_1 \gamma_w + H_2 \gamma_{sat}) - (H_1 + H_2 - h) \gamma_w \]

\[ \sigma'_B = H_2 (\gamma_{sat} - \gamma_w) + h \gamma_w \]

\[ \sigma'_B = H_2 \gamma'_w + h \gamma_w \]

---

*Figure 6.4a. Das FGE (2005).*
Figure 6.4a. Das FGE (2005).

**Effective Stress Concept**

**Stresses @ Point C:**

\[
\sigma_C = H_1 \gamma_w + z \gamma_{sat}
\]

\[
\left( \frac{h}{H_2} \right) u_C = (H_1 + z - \frac{h}{H_2} z) \gamma_w
\]

\[
\sigma'_C = \sigma_C - u_C
\]

\[
\sigma'_C = H_1 \gamma_w + z \gamma_{sat} - (H_1 + z - \frac{h}{H_2} z) \gamma_w
\]

\[
\sigma'_C = z \gamma' + \frac{h}{H_2} z \gamma_w
\]

**NOTE:**

\[
i = \frac{h}{H_2} = \frac{\text{Change in Head}}{\text{Length of Water Flow}}
\]

\[
\therefore \sigma'_C = z \gamma' + iz \gamma_w
\]
EFFECTIVE STRESS CONCEPT

DOWNWARD SEEPAGE

Figure 6.4b. Das FGE (2005).
EFFECTIVE STRESS CONCEPT
PARTIALLY SATURATED SOIL

\[ \sigma' = \sigma - u_a + \chi (u_a - u_w) \]

Where:
- \( u_a \) = Pore Air Pressure
- \( u_w \) = Pore Water Pressure
- \( \chi \) = Fraction of unit cross-sectional area of soil occupied by water.
  - \( \chi = 0 \) for dry soil; 1 for saturated soil.
- \( \chi \) depends on degree of saturation (S). Also influenced by soil structure.

Figure 6.6. Das FGE (2005).
**CAPILLARY RISE IN SOILS**

Summing Forces in Vertical Direction

\[
\frac{\pi}{4} d^2 h_c \gamma_w = \pi d T \cos \alpha
\]

\[
h_c = \frac{4T \cos \alpha}{d \gamma_w}
\]

Where:

- \(T\) = Surface Tension
- \(\alpha\) = Angle of Contact
- \(d\) = Capillary Tube Diameter

**Figure 8.19.** Principles of Geotechnical Engineering, Das (2006).

\[h_c \propto \frac{1}{d}\]
CAPILLARY RISE IN SOILS

Figure 8.20. Principles of Geotechnical Engineering, Das (2006).
CAPILLARY RISE IN SOILS

Hazen (1930)

\[ h_1 = \frac{C}{eD_{10}} \]

Where:

- \( D_{10} \) = Effective Size (mm)
- \( e \) = Void Ratio
- \( C \) = Constant (ranging from 10 mm\(^2\) to 50 mm\(^2\))
## CAPILLARY RISE IN SOILS

**Table 8.2** (Das, PGE 2006). Approximate Range of Capillary Rise in Soils.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Range of Capillary Rise</th>
<th>m</th>
<th>ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse Sand</td>
<td>0.1 – 0.2</td>
<td>0.3 – 0.6</td>
<td></td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.3 – 1.2</td>
<td>1 – 4</td>
<td></td>
</tr>
<tr>
<td>Silt</td>
<td>0.75 – 7.5</td>
<td>2.5 – 25</td>
<td></td>
</tr>
<tr>
<td>Clay</td>
<td>7.5 - 23</td>
<td>25 - 75</td>
<td></td>
</tr>
</tbody>
</table>

### EFFECTIVE STRESSES IN CAPILLARY ZONE

\[
\sigma' = \sigma - u
\]

- **Saturated:**  
  \[
  u = -h \gamma_w
  \]

- **Partially Saturated:**  
  \[
  u = -h \left( \frac{S}{100} \right) \gamma_w
  \]
**SEEPAGE FORCE**

**WITH NO SEEPAGE**
(i.e. STATIC CONDITIONS)

\[
\sigma' = (H_A - H)(\gamma_{sat} - \gamma_w)
\]

\[
(\gamma_{sat} - \gamma_w) = \gamma'
\]

Height of soil column = \(z\)

\(\gamma' = \) Submerged unit weight of soil

**Effective Stress:**
\[
\sigma' = z\gamma'
\]

**Effective Force:**
\[
P'_1 = z\gamma' A
\]

Where:
\(A = \text{Area}\)

---

Figure 6.1. Das FGE (2005).
SEEPAGE FORCE – UPWARD SEEPAGE

**EFFECTIVE STRESS**

- NO SEEPAGE:
  \[ \sigma' = z\gamma' \]

- W/ SEEPAGE:
  \[ \sigma' = z\gamma' - iz\gamma_w \]

**EFFECTIVE FORCE**

- NO SEEPAGE:
  \[ P'_1 = z\gamma'A \]

- W/ SEEPAGE:
  \[ P'_2 = (z\gamma' - iz\gamma_w)A \]

**DECREASE OF TOTAL FORCE DUE TO SEEPAGE:**

\[ P'_1 - P'_2 = iz\gamma_wA \]

**SEEPAGE FORCE PER UNIT VOLUME:**

\[ \frac{P'_1 - P'_2}{\text{(Soil Volume)}} = \frac{iz\gamma_wA}{zA} = iz\gamma_w \]
SEEPAGE FORCE SUMMARY

**NO SEEPAGE**

\[ \gamma' A \]

Volume of soil = \( zA \)

**UPWARD SEEPAGE**

\[ (\gamma' + i\gamma_w)A \]

\[ iz\gamma_w A = \text{seepage force} \]

**DOWNWARD SEEPAGE**

\[ (\gamma' - i\gamma_w)A \]

\[ iz\gamma_w A = \text{seepage force} \]