BERNOULLI’S EQUATION

\[ h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z \]

Where:

- \( h \) = Total Head
- \( u \) = Pressure
- \( v \) = Velocity
- \( g \) = Acceleration due to Gravity
- \( \gamma_w \) = Unit Weight of Water
BERNOULLI’S EQUATION IN SOIL

\[ h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z \]

\( \gamma \approx 0 \) (i.e. velocity of water in soil is negligible).

Therefore:

\[ h = \frac{u}{\gamma_w} + Z \]
CHANGE IN HEAD FROM POINTS A & B ($\Delta h$)

$$\Delta h = h_A - h_B$$

$\Delta h$ can be expressed in non-dimensional form

$$i = \frac{\Delta h}{L}$$

Where:

$i$ = Hydraulic Gradient

$L$ = Length of Flow between Points A & B
VELOCITY ($v$) vs. HYDRAULIC GRADIENT ($i$)

General relationship shown in Figure 5.2

Three Zones:
1. Laminar Flow (I)
2. Transition Flow (II)
3. Turbulent Flow (III)

For most soils, flow is laminar. Therefore:

$v \propto i$

Figure 5.2. Das FGE (2005).
DARCY’S LAW (1856)

\[ v = ki \]

Where:

- \( v \) = Discharge Velocity (i.e. quantity of water in unit time through unit cross-sectional area at right angles to the direction of flow)
- \( k \) = Hydraulic Conductivity (i.e. coefficient of permeability)
- \( i \) = Hydraulic Gradient

* Based on observations of flow of water through clean sands
HYDRAULIC CONDUCTIVITY ($k$)

$$k = \frac{\gamma_w}{\eta} \bar{K}$$

Where:

$\eta =$ Viscosity of Water

$\bar{K} =$ Absolute Permeability (units of $L^2$)

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$k$ (cm/sec)</th>
<th>$k$ (ft/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Gravel</td>
<td>100-1</td>
<td>200-2</td>
</tr>
<tr>
<td>Coarse Sand</td>
<td>1-0.01</td>
<td>2-0.02</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.01-0.001</td>
<td>0.02-0.002</td>
</tr>
<tr>
<td>Silty Clay</td>
<td>0.001-0.0000</td>
<td>0.002-0.0000</td>
</tr>
<tr>
<td>Clay</td>
<td>&lt; 0.000001</td>
<td>&lt;0.000002</td>
</tr>
</tbody>
</table>

after Table 5.1. Das FGE (2005)
**DISCHARGE & SEEPAGE VELOCITIES**

\[ q = vA = A_v v_s \]

Where:
- \( q \) = Flow Rate  
  (quantity of water/unit time)
- \( A \) = Total Cross-sectional Area
- \( A_v \) = Area of Voids
- \( v_s \) = Seepage Velocity

**Figure 5.3.** Das FGE (2005).
DISCHARGE & SEEPAGE VELOCITIES

\[ \begin{align*}
q &= v(A_v + A_s) = A_v v_s \\

v_s &= \frac{v(A_v + A_s)}{A_v} = \frac{v(A_v + A_s)}{A_v L} \\

v_s &= \frac{v(V_v + V_s)}{V_v} = v \left[ 1 + \left( \frac{V_v}{V_s} \right) \right] = \frac{v}{n} \\

\end{align*} \]

Figure 5.3. Das FGE (2005).
EXAMPLE PROBLEM

GIVEN:

- **CL (Impervious Layer)**
- **SM (k = 0.007 ft/min)**
- **175 ft**
- **25 ft**
- **12 ft**
- **15 ft**

REQUIRED:

Find Hydraulic Gradient \( (i) \) and Flow Rate \( (q) \)
EXAMPLE PROBLEM – FIND $i$, $q$

GIVEN:

- $k = 0.007$ ft/min
- $H_1 = 25$ ft
- $H_2 = 15$ ft
- $L = 175$ ft
- $\Delta h = 12$ ft
- $\theta = 12^\circ$

SOLUTION:

Hydraulic Gradient ($i$):

$$i = \frac{\Delta h}{L} = \frac{12}{175} \left(\frac{ft}{175 ft} \right) = 0.067$$

Rate of Flow per Time ($q$):

$$q = kiA$$

$$q = 0.007 \frac{ft}{min} (0.067)(15 ft)(\cos 12^\circ)(1 ft)$$

$$q = 6.9 \times 10^{-3} \frac{ft^3}{min/ft}$$
LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

Constant Head (ASTM D2434)

Falling Head (no ASTM)

Figure 5.4. Das FGE (2005).

Figure 5.5. Das FGE (2005).
LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

Constant Head
(ASTM D2434)

\[ Q = Avt = A(ki)t \]

Where:
- \( Q \) = Quantity of water collected over time \( t \)
- \( t \) = Duration of water collection

\[ Q = A \left( k \frac{h}{L} \right) t \]

\[ k = \frac{QL}{Aht} \]

Figure 5.4. Das FGE (2005).
LABORATORY TESTING OF HYDRAULIC CONDUCTIVITY

Falling Head
(No ASTM)

\[ q = k \frac{h}{L} A = -a \frac{dh}{dt} \]

Where:

- \( A \) = Cross-sectional area of Soil
- \( a \) = Cross-sectional area of Standpipe

after rearranging above equation

\[ dt = \frac{aL}{Ak} \left( -\frac{dh}{h} \right) \]

Integrate from limits 0 to \( t \)

after integration

\[ t = \frac{aL}{Ak} \log e \frac{h_1}{h_2} \quad \text{or} \quad k = 2.303 \frac{aL}{At} \log_{10} \frac{h_1}{h_2} \]

Figure 5.5. Das FGE (2005).
EMPIRICAL RELATIONSHIPS FOR HYDRAULIC CONDUCTIVITY

**Uniform Sands - Hazen Formula**
(Hazen, 1930):

\[ k (cm/\sec) = c D_{10}^2 \]

Where:
- \( c \) = Constant between 1 to 1.5
- \( D_{10} \) = Effective Size (in mm)

**Sands – Kozeny-Carman**
(Loudon 1952 and Perloff and Baron 1976):

\[ k = C_1 \frac{e^3}{1 + e} \]

Where:
- \( C_1 \) = Constant (to be determined) 
- \( e \) = Void Ratio

**Sands – Casagrande**
(Unpublished):

\[ k = 1.4 e^2 k_{0.85} \]

Where:
- \( e \) = Void Ratio
- \( k_{0.85} \) = Hydraulic Conductivity @ \( e = 0.85 \)

**Normally Consolidated Clays**
(Samarasinghe, Huang, and Drnevich, 1982):

\[ k = C_2 \left( \frac{e^n}{1 + e} \right) \]

Where:
- \( C_2 \) = Constant to be determined experimentally
- \( n \) = Constant to be determined experimentally
- \( e \) = Void Ratio
EXAMPLE - ESTIMATION OF HYDRAULIC CONDUCTIVITY (NORMALLY CONSOLIDATED CLAYS)

**GIVEN:**
Normally consolidated clay with \( e \) and \( k \) measurements from 1D Consolidation Test.

<table>
<thead>
<tr>
<th>Void Ratio (e)</th>
<th>( k ) (cm/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>( 0.6 \times 10^{-7} )</td>
</tr>
<tr>
<td>1.52</td>
<td>( 1.52 \times 10^{-7} )</td>
</tr>
</tbody>
</table>

**REQUIRED:**
Find \( k \) for same clay with a void ratio of 1.4.

**SOLUTION:**
Using (Samarasinghe, Huang, and Drnevich, 1982) Equation:

\[
\frac{k_1}{k_2} = \frac{C_2 \left( \frac{e_1^n}{1 + e_1} \right)}{C_2 \left( \frac{e_2^n}{1 + e_2} \right)}
\]

Substituting known quantities:

\[
\frac{0.6 \text{ cm/sec}}{1.52 \text{ cm/sec}} = \left( \frac{1.2}{1.52} \right)^n \left( \frac{2.52}{2.2} \right)
\]

Example 5.6 Das PGE (2005).
EXAMPLE - ESTIMATION OF HYDRAULIC CONDUCTIVITY (NORMALLY CONSOLIDATED CLAYS) (continued)

\[
\frac{0.6 \text{ cm/sec}}{1.52 \text{ cm/sec}} = \left( \frac{1.2}{1.52} \right)^n \left( \frac{2.52}{2.2} \right) \therefore n = 4.5
\]

\[
k_1 = C_2 \left( \frac{e_1^n}{1 + e_1} \right)
\]

\[
0.6 \times 10^{-7} \text{ cm/sec} = \left( \frac{1.2^{4.5}}{1 + 1.2} \right)
\]

\[
\therefore C_2 = 0.581 \times 10^{-7} \text{ cm/sec}
\]

\[
k_{e=1.4} = (0.581 \times 10^{-7} \text{ cm/sec}) \left( \frac{1.4^{4.5}}{1 + 1.4} \right) = 1.1 \times 10^{-7} \text{ cm/sec}
\]

Example 5.6 Das PGE (2005).
EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – HORIZONTAL DIRECTION

Considering cross-section of Unit Length 1.

Total flow through cross-section can be written as:

\[ q = v \cdot 1 \cdot H \]

\[ q = v_1 \cdot 1 \cdot H_1 + v_2 \cdot 1 \cdot H_2 + \ldots + v_n \cdot 1 \cdot H_n \]

Where:

\( v = \) Average Discharge Velocity

\( v_1 = \) Discharge Velocity in Layer 1

Figure 5.7. Das FGE (2005).
EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – HORIZONTAL DIRECTION

Substituting $\nu = k_i$ into $q$ equation and using $H$ to denote Horizontal Direction

$v = k_{H(eq)} i_{eq}$

$v_1 = k_{H_1} i_1; v_2 = k_{H_2} i_2; \ldots; v_n = k_{H_n} i_n$

Noting that $i_{eq} = i_1 = i_2 = \ldots = i_n$

$k_{H(eq)} = \frac{1}{H} (k_{H_1} H_1 + k_{H_2} H_2 + \ldots + k_{H_n} H_n)$

Where $k_{H(eq)} = \text{Equivalent Hydraulic Conductivity in Horizontal Direction}$

Figure 5.7. Das FGE (2005).
EQUIVALENT HYDRAULIC CONDUCTIVITY IN STRATIFIED SOILS – VERTICAL DIRECTION

Total Head Loss = $h$
$h =$ Sum Head Loss in Each Layer

$$v = v_1 = v_2 = \ldots = v_n$$

and

$$h = h_1 = h_2 = \ldots = h_n$$

Using Darcy’s Law ($v=ki$) into $v$ equation and using $V$ to denote Vertical Direction

$$k_{V(eq)} \frac{h}{H} = k_{V1}i_1 = \ldots = k_{Vn}i_n$$

Where $k_{V(eq)}$ = Equivalent Hydraulic Conductivity in Vertical Direction

Figure 5.8. Das FGE (2005).
FIELD PERMEABILITY TEST BY PUMPING WELLS
UNCONFINED PERMEABLE LAYER UNDERLAIN BY IMPERMEABLE LAYER

q = Groundwater Flow into Well
q also is rate of discharge from pumping

Field Measurements Taken:
q, r₁, r₂, h₁, h₂

Equation:

\[ q = k \left( \frac{dh}{dr} \right) 2\pi r h \]

can be re-written as

\[ \int_{r_2}^{r_1} \frac{dr}{r} = \left( \frac{2\pi k}{q} \right) \int_{h_2}^{h_1} h dh \]

Solving Equation:

\[ k_{field} = \frac{2.303q \log_{10} \left( \frac{r_1}{r_2} \right)}{\pi \left( h_1^2 - h_2^2 \right)} \]

Figure 5.9. Das FGE (2005).
FIELD PERMEABILITY TEST BY PUMPING WELLS
WELL PENETRATING CONFINED AQUIFER

$q = \text{Groundwater Flow into Well}
q$ also is rate of discharge from pumping

Equation:
\[ q = k \left( \frac{dh}{dr} \right) 2\pi r H \]

can be re-written as
\[ \int_{r_2}^{r_1} \frac{dr}{r} = \int_{h_2}^{h_1} \frac{2\pi k H}{q} dh \]

Solving Equation:
\[ k_{field} = \frac{q \log_{10} \left( \frac{r_1}{r_2} \right)}{2.727 H (h_1 - h_2)} \]

Field Measurements Taken:
$q, r_1, r_2, h_1, h_2$
SOIL PERMEABILITY & DRAINAGE

Coefficient of Permeability $k$ (cm/sec) (log scale)

<table>
<thead>
<tr>
<th>DRAINAGE</th>
<th>Good</th>
<th>Poor</th>
<th>Practically Impervious</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Gravel</td>
<td>Clean Sands, Clean Sand &amp; Gravel Mixtures</td>
<td>Very Fine Sands; Organic &amp; Inorganic Silt; Mixtures of Sand, Silt, and Clay; Till; Stratified Clay Deposits, etc.</td>
<td>&quot;Impervious&quot; Soils (e.g. clays)</td>
</tr>
<tr>
<td>Direct Testing of Soil in its Original Position - Pumping Tests; Reliable if Properly Conducted; Considerable Experience Required</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Head Permeameter; Little Experience Required</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INDIRECT DETERMINATION OF $k$</td>
<td>Falling Head Permeameter; Reliable; Little Experience Required</td>
<td>Falling Head Permeameter; Unreliable; Much Experience Required</td>
<td>Falling Head Permeameter; Reliable; Considerable Experience Necessary</td>
</tr>
<tr>
<td>Computation from Grain Size Distribution Applicable Only to Clean Cohesionless Sands and Gravel</td>
<td>Computation Based on 1D Consolidation Tests; Reliable; Considerable Experience Required</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

after Casagrande and Fadum (1940) and Terzaghi et al. (1996).
### SOIL PERMEABILITY & DRAINAGE

#### COEFFICIENT OF PERMEABILITY

<table>
<thead>
<tr>
<th>CM/S (LOG SCALE)</th>
<th>10^2</th>
<th>10^1</th>
<th>1.0</th>
<th>10^-1</th>
<th>10^-2</th>
<th>10^-3</th>
<th>10^-4</th>
<th>10^-5</th>
<th>10^-6</th>
<th>10^-7</th>
<th>10^-8</th>
<th>10^-9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage property</td>
<td>Good drainage</td>
<td>Poor drainage</td>
<td>Practically impervious</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Application in earth dams and dikes</td>
<td>Pervious sections of dams and dikes</td>
<td>Impervious sections of earth dams and dikes</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of soil</td>
<td>Clean gravel</td>
<td>Clean sands, clean sand and gravel mixtures</td>
<td>Very fine sands, organic and inorganic silts, mixtures of sand, silt, and clay glacial till, stratified clay deposits, etc.</td>
<td>&quot;Impervious&quot; soils which are modified by the effect of vegetation and weathering, fissured, weathered clays; fractured OC clays</td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct determination of coefficient of permeability</td>
<td>Direct testing of soil in its original position (e.g., well points). Properly conducted, reliable, considerable experience required.</td>
<td>(Note: Considerable experience also required in this range.)</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Constant Head Permeameter; little experience required.</td>
<td>Fairly reliable; considerable experience necessary (do in triaxial cell)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Reliably, little experience required</td>
<td>Reliably, range of unstable permeability; much experience necessary to correct interpretation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Indirect determination of coefficient of permeability</td>
<td>Computation: From the grain size distribution (e.g., Hazen’s formula). Only applicable to clean, cohesionless sands and gravels</td>
<td>Horizontal Capillary Test: Very little experience necessary; especially useful for rapid testing of a large number of samples in the field without laboratory facilities.</td>
<td>Computation: from consolidation tests; expensive laboratory equipment and considerable experience required.</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Due to migration of fines, channels, and air in voids.*

---

From FHWA IF-02-034 *Evaluation of Soil and Rock Properties.*
HYDRAULIC CONDUCTIVITY & SEEPAGE

LAPLACE’S EQUATION OF CONTINUITY

Steady-State Flow around an impervious Sheet Pile Wall

Consider water flow at Point A:

\[ v_x = \text{Discharge Velocity in } x \text{ Direction} \]

\[ v_z = \text{Discharge Velocity in } z \text{ Direction} \]

Y Direction Out Of Plane

Figure 5.11. Das FGE (2005).
LAPLACE’S EQUATION OF CONTINUITY

Consider water flow at Point A (Soil Block at Pt A shown left)

Rate of water flow into soil block in x direction:
\[ v_x \, dz \, dy \]

Rate of water flow into soil block in x direction:
\[ v_z \, dx \, dy \]

Rate of water flow out of soil block in x,z directions:
\[
\left( v_x + \frac{\partial v_x}{\partial x} \, dx \right) dz \, dy
\]
\[
\left( v_z + \frac{\partial v_z}{\partial z} \, dz \right) dx \, dy
\]
LAPLACE’S EQUATION OF CONTINUITY

Consider water flow at Point A (Soil Block at Pt A shown left)

Total Inflow = Total Outflow

\[
\left[ \left( v_x + \frac{\partial v_x}{\partial x} \right) dx dy \right] dz dy + \left[ \left( v_z + \frac{\partial v_z}{\partial z} \right) dx dy \right] dz dy - \left[ v_x dz dy + v_z dx dy \right] = 0
\]

or

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} = 0
\]

Figure 5.11. Das FGE (2005).
LAPLACE’S EQUATION OF CONTINUITY

Consider water flow at Point A (Soil Block at Pt A shown left)

Using Darcy’s Law \(v = ki\)

\[
v_x = k_x i_x = k_x \left( -\frac{\partial h}{\partial x} \right)
\]

\[
v_z = k_z i_z = k_z \left( -\frac{\partial h}{\partial z} \right)
\]

\[
\therefore \quad k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0
\]

Figure 5.11. Das FGE (2005).
FLOW NETS – DEFINITION OF TERMS


Flow Line: A line along which a water particle moves through a permeable soil medium.

Flow Channel: Strip between any two adjacent Flow Lines.

Equipotential Lines: A line along which the potential head at all points is equal.

NOTE: Flow Lines and Equipotential Lines must meet at right angles!
FLOW NETS

FLOW AROUND SHEET PILE WALL

Figure 5.12a. Das FGE (2005).

Flow line $k_x = k_z = k$

Equipotential line

Impervious layer

Sheet pile
FLOW NETS

FLOW AROUND SHEET PILE WALL

\[ k_x = k_z = k \]
\[ N_f = 4 \]
\[ N_d = 6 \]

Figure 5.12b. Das FGE (2005).
FLOW NETS – BOUNDARY CONDITIONS

1. The upstream and downstream surfaces of the permeable layer (i.e. lines $ab$ and $de$ in Figure 12b Das FGE (2005)) are equipotential lines.

2. Because $ab$ and $de$ are equipotential lines, all the flow lines intersect them at right angles.

3. The boundary of the impervious layer (i.e. line $fg$ in Figure 12b Das FGE (2005)) is a flow line, as is the surface of the impervious sheet pile (i.e. line $acd$ in Figure 12b Das FGE (2005)).

4. The equipotential lines intersect $acd$ and $fg$ (Figure 12b Das FGE (2005)) at right angles.
FLOW NETS

Flow under a Impermeable Dam

$k_x = k_z = k$
$N_f = 4$
$N_d = 8$

Figure 5.13. Das FGE (2005).
Rate of Seepage Through Flow Channel (per unit length):

\[ \Delta q_1 = \Delta q_2 = \Delta q_3 = \ldots = \Delta q_n \]

Using Darcy’s Law
\( q = vA = kiA \)

\[ \Delta q = k \left( \frac{h_1 - h_2}{l_1} \right) l_1 = k \left( \frac{h_2 - h_3}{l_2} \right) l_2 = k \left( \frac{h_3 - h_4}{l_3} \right) l_3 = \ldots \]

Potential Drop
\[ h_1 - h_2 = h_2 - h_3 = h_3 - h_4 = \ldots = \frac{H}{N_d} \]

Where:
- \( H \) = Head Difference
- \( N_d \) = Number of Potential Drops

Figure 5.14. Das FGE (2005).
Therefore, flow through one channel is:

$$\Delta q = k \frac{H}{N_d}$$

If Number of Flow Channels = $N_f$, then the total flow for all channels per unit length is:

$$q = k \frac{HN_f}{N_d}$$

$N_f = 4$

$N_d = 6$

Figure 5.12b. Das FGE (2005).
GIVEN:
Flow Net in Figure 5.17.
N_f = 3
N_d = 6
k_x = k_z = 5 \times 10^{-3} \text{ cm/sec}

DETERMINE:

a. How high water will rise in piezometers at points a, b, c, and d.
b. Rate of seepage through flow channel II.
c. Total rate of seepage.
FLOW NETS
FLOW AROUND SHEET PILE WALL EXAMPLE

SOLUTION:

Potential Drop = \[ \frac{H}{N_d} \]

\[ \frac{(5m - 1.67m)}{6} = 0.56m \]

At Pt a:
Water in standpipe = (5m – 1x0.56m) = 4.44m

At Pt b:
Water in standpipe = (5m – 2x0.56m) = 3.88m

At Pts c and d:
Water in standpipe = (5m – 5x0.56m) = 2.20m

Figure 5.17. Das FGE (2005).
FLOW NETS

FLOW AROUND SHEET PILE WALL EXAMPLE

SOLUTION:

\[ \Delta q = k \frac{H}{N_d} \]

\[ k = 5 \times 10^{-3} \text{ cm/sec} = 5 \times 10^{-5} \text{ m/sec} \]

\[ \Delta q = (5 \times 10^{-5} \text{ m/sec})(0.56 \text{ m}) \]

\[ \Delta q = 2.8 \times 10^{-5} \text{ m}^3/\text{sec}/\text{m} \]

\[ q = k \frac{HN_f}{N_d} = \Delta q N_f \]

\[ q = (2.8 \times 10^{-5} \text{ m}^3/\text{sec}/\text{m}) \times 3 \]

\[ q = 8.4 \times 10^{-5} \text{ m}^3/\text{sec}/\text{m} \]

Figure 5.17. Das FGE (2005).