PROBLEM 3.70

Two 80-N forces are applied as shown to the corners $B$ and $D$ of a rectangular plate. (a) Determine the moment of the couple formed by the two forces by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples. (b) Use the result obtained to determine the perpendicular distance between lines $BE$ and $DF$.

SOLUTION

(a) Resolving forces into components:

$$P = (80 \text{ N}) \sin 50^\circ = 61.284 \text{ N}$$

$$Q = (80 \text{ N}) \cos 50^\circ = 51.423 \text{ N}$$

$$M = (51.423 \text{ N})(0.5 \text{ m}) - (61.284 \text{ N})(0.3 \text{ m})$$

$$= 7.3263 \text{ N} \cdot \text{m}$$

\[ M = 7.33 \text{ N} \cdot \text{m} \]

(b) Distance between lines $BE$ and $DF$

$$M = Fd$$

or

$$7.3263 \text{ N} \cdot \text{m} = (80 \text{ N})d$$

$$d = 0.091579 \text{ m}$$

$$d = 91.6 \text{ mm}$$
**PROBLEM 3.72**

Four 1 1/2-in.-diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. (a) Determine the resultant couple acting on the board. (b) If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string? (c) What is the value of that minimum tension?

---

**SOLUTION**

(a) \[ M = (60 \text{ lb})(10.5 \text{ in.}) + (40 \text{ lb})(13.5 \text{ in.}) \]
\[ = 630 \text{ lb \cdot in.} + 540 \text{ lb \cdot in.} \]
\[ M = 1170 \text{ lb \cdot in.} \]

(b) With only one string, pegs A and D, or B and C should be used. We have
\[ \tan \theta = \frac{9}{12} \]
\[ \theta = 36.9^\circ \quad 90^\circ - \theta = 53.1^\circ \]

Direction of forces:
With pegs A and D: \[ \theta = 53.1^\circ \]
With pegs B and C: \[ \theta = 53.1^\circ \]

(c) The distance between the centers of the two pegs is
\[ \sqrt{12^2 + 9^2} = 15 \text{ in.} \]

Therefore, the perpendicular distance \( d \) between the forces is
\[ d = 15 \text{ in.} + 2 \left( \frac{3}{4} \text{ in.} \right) \]
\[ = 16.5 \text{ in.} \]

We must have
\[ M = Fd \quad 1170 \text{ lb \cdot in.} = F(16.5 \text{ in.}) \]
\[ F = 70.9 \text{ lb} \]
PROBLEM 3.74

A piece of plywood in which several holes are being drilled successively has been secured to a workbench by means of two nails. Knowing that the drill exerts a 12-N·m couple on the piece of plywood, determine the magnitude of the resulting forces applied to the nails if they are located (a) at A and B, (b) at B and C, (c) at A and C.

SOLUTION

(a) \[ M = Fd \]
\[ 12 \text{ N} \cdot \text{m} = F(0.45 \text{ m}) \]
\[ F = 26.7 \text{ N} \]

(b) \[ M = Fd \]
\[ 12 \text{ N} \cdot \text{m} = F(0.24 \text{ m}) \]
\[ F = 50.0 \text{ N} \]

(c) \[ M = Fd \]
\[ d = \sqrt{(0.45 \text{ m})^2 + (0.24 \text{ m})^2} = 0.510 \text{ m} \]
\[ 12 \text{ N} \cdot \text{m} = F(0.510 \text{ m}) \]
\[ F = 23.5 \text{ N} \]
PROBLEM 3.78

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Replace the couple in the $ABCD$ plane with two couples $P$ and $Q$ shown:

$P = (50 \text{ N}) \frac{CD}{CG} = (50 \text{ N}) \left( \frac{160 \text{ mm}}{200 \text{ mm}} \right) = 40 \text{ N}$

$Q = (50 \text{ N}) \frac{CF}{CG} = (50 \text{ N}) \left( \frac{120 \text{ mm}}{200 \text{ mm}} \right) = 30 \text{ N}$

Couple vector $\mathbf{M}_1$ perpendicular to plane $ABCD$:

$\mathbf{M}_1 = (40 \text{ N})(0.24 \text{ m}) - (30 \text{ N})(0.16 \text{ m}) = 4.80 \text{ N} \cdot \text{m}$

Couple vector $\mathbf{M}_2$ in the $xy$ plane:

$\mathbf{M}_2 = -(12.5 \text{ N})(0.192 \text{ m}) = -2.40 \text{ N} \cdot \text{m}$

$\tan \theta = \frac{144 \text{ mm}}{192 \text{ mm}} \quad \theta = 36.870^\circ$

$\mathbf{M}_1 = (4.80 \cos 36.870^\circ) \mathbf{j} + (4.80 \sin 36.870^\circ) \mathbf{k}$

$= 3.84 \mathbf{j} + 2.88 \mathbf{k}$

$\mathbf{M}_2 = -2.40 \mathbf{j}$

$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = 1.44 \mathbf{j} + 2.88 \mathbf{k}$

$\mathbf{M} = 3.22 \text{ N} \cdot \text{m}; \; \theta_x = 90.0^\circ, \; \theta_y = 53.1^\circ, \; \theta_z = 36.9^\circ$