PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: \( R = 1391 \text{ kN}, \quad \alpha = 47.8^\circ \)
PROBLEM 2.3

Two structural members $B$ and $C$ are bolted to bracket $A$. Knowing that both members are in tension and that $P = 10$ kN and $Q = 15$ kN, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: $R = 20.1$ kN, $\alpha = 21.2^\circ$
PROBLEM 2.7

A telephone cable is clamped at $A$ to the pole $AB$. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000$ lb, determine by trigonometry $(a)$ the required tension $T_1$ in the left-hand portion if the resultant $R$ of the forces exerted by the cable at $A$ is to be vertical, $(b)$ the corresponding magnitude of $R$.

SOLUTION

Using the triangle rule and the law of sines:

$(a)$

$$75^\circ + 40^\circ + \beta = 180^\circ$$

$$\beta = 180^\circ - 75^\circ - 40^\circ = 65^\circ$$

$$\frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{T_1}{\sin 65^\circ}$$

$$T_1 = 938 \text{ lb}$$

$(b)$

$$\frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

$$R = 665 \text{ lb}$$
**PROBLEM 2.11**

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20^\circ$, determine by trigonometry

(a) the required magnitude of the force \( P \) if the resultant \( R \) of the two forces applied at \( A \) is to be vertical, 

(b) the corresponding magnitude of \( R \).

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**SOLUTION**

Using the triangle rule and the law of sines:

(a) \[ \beta + 50^\circ + 60^\circ = 180^\circ \]
\[ \beta = 180^\circ - 50^\circ - 60^\circ \]
\[ = 70^\circ \]
\[ \frac{425 \text{ lb}}{\sin 70^\circ} = \frac{P}{\sin 60^\circ} \]
\[ P = 392 \text{ lb} \]

(b) \[ \frac{425 \text{ lb}}{\sin 70^\circ} = \frac{R}{\sin 50^\circ} \]
\[ R = 346 \text{ lb} \]
PROBLEM 2.22

Determine the $x$ and $y$ components of each of the forces shown.

SOLUTION

Compute the following distances:

\[ OA = \sqrt{(600)^2 + (800)^2} = 1000 \text{ mm} \]
\[ OB = \sqrt{(560)^2 + (900)^2} = 1060 \text{ mm} \]
\[ OC = \sqrt{(480)^2 + (900)^2} = 1020 \text{ mm} \]

800-N Force:

\[ F_x = +(800 \text{ N}) \frac{800}{1000} \quad F_x = +640 \text{ N} \]
\[ F_y = +(800 \text{ N}) \frac{600}{1000} \quad F_y = +480 \text{ N} \]

424-N Force:

\[ F_x = -(424 \text{ N}) \frac{560}{1060} \quad F_x = -224 \text{ N} \]
\[ F_y = -(424 \text{ N}) \frac{900}{1060} \quad F_y = -360 \text{ N} \]

408-N Force:

\[ F_x = +(408 \text{ N}) \frac{480}{1020} \quad F_x = +192.0 \text{ N} \]
\[ F_y = -(408 \text{ N}) \frac{900}{1020} \quad F_y = -360 \text{ N} \]
PROBLEM 2.31

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.21:

<table>
<thead>
<tr>
<th>Force</th>
<th>x Comp. (lb)</th>
<th>y Comp. (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 lb</td>
<td>+21.0</td>
<td>+20.0</td>
</tr>
<tr>
<td>50 lb</td>
<td>-14.00</td>
<td>+48.0</td>
</tr>
<tr>
<td>51 lb</td>
<td>+24.0</td>
<td>-45.0</td>
</tr>
</tbody>
</table>

\[ R_x = +31.0 \quad R_y = +23.0 \]

\[ R = R_x \mathbf{i} + R_y \mathbf{j} \]
\[ = (31.0 \text{ lb}) \mathbf{i} + (23.0 \text{ lb}) \mathbf{j} \]
\[ \tan \alpha = \frac{R_y}{R_x} \]
\[ = \frac{23.0}{31.0} \]
\[ \alpha = 36.573^\circ \]
\[ R = \frac{23.0 \text{ lb}}{\sin(36.573^\circ)} \]
\[ = 38.601 \text{ lb} \]

\[ R = 38.6 \text{ lb} \angle 36.6^\circ \]
PROBLEM 2.36
Knowing that the tension in rope $AC$ is 365 N, determine the resultant of the three forces exerted at point $C$ of post $BC$.

SOLUTION
Determine force components:

Cable force $AC$:

- $F_x = -\frac{(365 \text{ N}) \times 960}{1460} = -240 \text{ N}$
- $F_y = -(365 \text{ N}) \times \frac{1100}{1460} = -275 \text{ N}$

500-N Force:

- $F_x = (500 \text{ N}) \times \frac{24}{25} = 480 \text{ N}$
- $F_y = (500 \text{ N}) \times \frac{7}{25} = 140 \text{ N}$

200-N Force:

- $F_x = (200 \text{ N}) \times \frac{4}{5} = 160 \text{ N}$
- $F_y = -(200 \text{ N}) \times \frac{3}{5} = -120 \text{ N}$

and

- $R_x = \Sigma F_x = -240 \text{ N} + 480 \text{ N} + 160 \text{ N} = 400 \text{ N}$
- $R_y = \Sigma F_y = -275 \text{ N} + 140 \text{ N} - 120 \text{ N} = -255 \text{ N}$

- $R = \sqrt{R_x^2 + R_y^2}$
  - $R = \sqrt{(400 \text{ N})^2 + (-255 \text{ N})^2}$
  - $R = 474.37 \text{ N}$

Further:

- $\tan \alpha = \frac{255}{400}$
- $\alpha = 32.5^\circ$

$\mathbf{R} = 474 \text{ N} \angle 32.5^\circ$
PROBLEM 2.37

Knowing that $\alpha = 40^\circ$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force:  
$F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$  
$F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force:  
$F_x = (80 \text{ lb}) \cos 60^\circ = 40.000 \text{ lb}$  
$F_y = (80 \text{ lb}) \sin 60^\circ = 69.282 \text{ lb}$

120-lb Force:  
$F_x = (120 \text{ lb}) \cos 30^\circ = 103.923 \text{ lb}$  
$F_y = -(120 \text{ lb}) \sin 30^\circ = -60.000 \text{ lb}$

and  
$R_x = \Sigma F_x = 200.305 \text{ lb}$  
$R_y = \Sigma F_y = 29.803 \text{ lb}$  

$R = \sqrt{(200.305 \text{ lb})^2 + (29.803 \text{ lb})^2} = 202.510 \text{ lb}$

Further:  
$\tan \alpha = \frac{29.803}{200.305}$

$\alpha = \tan^{-1} \left( \frac{29.803}{200.305} \right) = 8.46^\circ$  

$R = 203 \text{ lb } \angle 8.46^\circ$
PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram

Force Triangle

Law of sines:
\[
\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{400 \text{ lb}}{\sin 80^\circ}
\]

(a) \hfill T_{AC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 60^\circ) \hfill T_{AC} = 352 \text{ lb} \\

(b) \hfill T_{BC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 40^\circ) \hfill T_{BC} = 261 \text{ lb}
PROBLEM 2.48
Knowing that $\alpha = 20^\circ$, determine the tension (a) in cable $AC$, (b) in rope $BC$.

SOLUTION

Free-Body Diagram

Force Triangle

Law of sines:

\[
\frac{T_{AC}}{\sin 110^\circ} = \frac{T_{BC}}{\sin 5^\circ} = \frac{1200 \text{ lb}}{\sin 65^\circ}
\]

(a)

\[
T_{AC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 110^\circ
\]

\[T_{AC} = 1244 \text{ lb} \quad \blacktriangleleft\]

(b)

\[
T_{BC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 5^\circ
\]

\[T_{BC} = 115.4 \text{ lb} \quad \blacktriangleleft\]
PROBLEM 2.49

Two cables are tied together at C and are loaded as shown. Knowing that \( P = 300 \, \text{N} \), determine the tension in cables \( AC \) and \( BC \).

SOLUTION

Free-Body Diagram

\[ \sum F_x = 0 \quad -T_{CA} \sin 30^\circ + T_{CB} \sin 30^\circ - P \cos 45^\circ - 200 \, \text{N} = 0 \]

For \( P = 200 \, \text{N} \) we have,

\[ -0.5T_{CA} + 0.5T_{CB} + 212.13 - 200 = 0 \quad (1) \]

\[ \sum F_y = 0 \quad T_{CA} \cos 30^\circ - T_{CB} \cos 30^\circ - P \sin 45^\circ = 0 \]

\[ 0.86603T_{CA} + 0.86603T_{CB} - 212.13 = 0 \quad (2) \]

Solving equations (1) and (2) simultaneously gives,

\[ T_{CA} = 134.6 \, \text{N} \]

\[ T_{CB} = 110.4 \, \text{N} \]
PROBLEM 2.67

A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley

(a) \[ + \sum F_y = 0: \quad 2T - (600 \text{ lb}) = 0 \]
   \[ T = \frac{1}{2}(600 \text{ lb}) \]
   \[ T = 300 \text{ lb} \]

(b) \[ + \sum F_y = 0: \quad 2T - (600 \text{ lb}) = 0 \]
   \[ T = \frac{1}{2}(600 \text{ lb}) \]
   \[ T = 300 \text{ lb} \]

(c) \[ + \sum F_y = 0: \quad 3T - (600 \text{ lb}) = 0 \]
   \[ T = \frac{1}{3}(600 \text{ lb}) \]
   \[ T = 200 \text{ lb} \]

(d) \[ + \sum F_y = 0: \quad 3T - (600 \text{ lb}) = 0 \]
   \[ T = \frac{1}{3}(600 \text{ lb}) \]
   \[ T = 200 \text{ lb} \]

(e) \[ + \sum F_y = 0: \quad 4T - (600 \text{ lb}) = 0 \]
   \[ T = \frac{1}{4}(600 \text{ lb}) \]
   \[ T = 150.0 \text{ lb} \]