PROBLEM 3.35

Given the vectors \( P = 2i + 3j - k \), \( Q = 5i - 4j + 3k \), and \( S = -3i + 2j - 5k \), compute the scalar products \( P \cdot Q \), \( P \cdot S \), and \( Q \cdot S \).

SOLUTION

\[
P \cdot Q = (2i + 3j - k) \cdot (5i - 4j + 3k) \\
= (2)(5) + (3)(-4) + (-1)(3) \\
= 10 - 12 - 3 \\
P \cdot Q = -5
\]

\[
P \cdot S = (2i + 3j - k) \cdot (-3i + 2j - 5k) \\
= (2)(-3) + (3)(2) + (-1)(-5) \\
= -6 + 6 + 5 \\
P \cdot S = +5
\]

\[
Q \cdot S = (5i - 4j + 3k) \cdot (-3i + 2j - 5k) \\
= (5)(-3) + (-4)(2) + (3)(-5) \\
= -15 - 8 - 15 \\
Q \cdot S = -38
\]
PROBLEM 3.40

Knowing that the tension in cable $AD$ is 180 lb, determine (a) the angle between cable $AD$ and the boom $AB$, (b) the projection on $AB$ of the force exerted by cable $AD$ at point $A$.

SOLUTION

(a) First note

\[ AD = \sqrt{(-6)^2 + (3)^2 + (-6)^2} = 9.00 \text{ ft} \]

\[ AB = \sqrt{(-6)^2 + (-4.5)^2 + (0)^2} = 7.50 \text{ ft} \]

and

\[ \overrightarrow{AD} = -(6 \text{ ft})\mathbf{i} + (3 \text{ ft})\mathbf{j} - (6 \text{ ft})\mathbf{k} \]

\[ \overrightarrow{AB} = -(6 \text{ ft})\mathbf{i} - (4.5 \text{ ft})\mathbf{j} \]

By definition,

\[ \overrightarrow{AD} \cdot \overrightarrow{AB} = (AD)(AB) \cos \theta \]

\[ (-6\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) \cdot (-6\mathbf{i} - 4.5\mathbf{j}) = (9.00)(7.50) \cos \theta \]

\[ (-6)(-6) + (3)(-4.5) + (-6)(0) = 67.50 \cos \theta \]

\[ \cos \theta = \frac{1}{3} \]

\[ \theta = 70.5^\circ \]

(b)

\[ (T_{AD})_{AB} = T_{AD} \cdot \lambda_{AB} = T_{AD} \cos \theta \]

\[ = (180 \text{ lb}) \left( \frac{1}{3} \right) \]

\[ (T_{AD})_{AB} = 60.0 \text{ lb} \]
PROBLEM 3.57

The frame $ACD$ is hinged at $A$ and $D$ and is supported by a cable that passes through a ring at $B$ and is attached to hooks at $G$ and $H$. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal $AD$ of the force exerted on the frame by portion $BH$ of the cable.

SOLUTION

$$M_{AD} = \lambda_{AD} \cdot (r_{B/A} \times T_{BH})$$

Where

$$\lambda_{AD} = \frac{1}{5}(4i - 3k)$$

$$r_{B/A} = (0.5 \text{ m})i$$

and

$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2} = 1.125 \text{ m}$$

Then

$$T_{BH} = \frac{450 \text{ N}}{1.125}(0.375i + 0.75j - 0.75k)$$

$$= (150 \text{ N})i + (300 \text{ N})j - (300 \text{ N})k$$

Finally,

$$M_{AD} = \frac{1}{5} \begin{bmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{bmatrix}$$

$$= \frac{1}{5}[(−3)(0.5)(300)]$$

or $M_{AD} = -90.0 \text{ N} \cdot \text{m}$