PROBLEM 12.66

An advanced spatial disorientation trainer allows the cab to rotate around multiple axes as well as extend inwards and outwards. It can be used to simulate driving, fixed wing aircraft flying, and helicopter maneuvering. In one training scenario, the trainer rotates and translates in the horizontal plane where the location of the pilot is defined by the relationships

\[ r = 10 + 2 \cos \left( \frac{\pi}{3} t \right) \] and \( \theta = 0.1 \left( 2t^2 - t \right) \), where \( r \), \( \theta \), and \( t \) are expressed in feet, radians, and seconds, respectively. Knowing that the pilot has a mass of 175 lbs, (a) find the magnitude of the resulting force acting on the pilot at \( t = 5 \) s (b) plot the magnitudes of the radial and transverse components of the force exerted on the pilot from 0 to 10 seconds.

SOLUTION

Given:

\[ r = 10 + 2 \cos \left( \frac{\pi}{3} t \right) \text{ ft} \]
\[ \theta = 0.1 \left( 2t^2 - t \right) \text{ rad} \]
\[ m = \frac{175 \text{ lbs}}{32.2 \text{ ft/s}^2} = 5.435 \text{ slugs} \]

Using Radial and Transverse Coordinates:

\[ \dot{r} = \frac{2\pi}{3} \sin \left( \frac{\pi}{3} t \right) \text{ ft/s} \]
\[ \ddot{r} = \frac{2\pi^2}{9} \cos \left( \frac{\pi}{3} t \right) \text{ ft/s}^2 \]
\[ \dot{\theta} = 0.1 \left( 4t - 1 \right) \text{ rad/s} \]
\[ \ddot{\theta} = 0.4 \text{ rad/s}^2 \]

Free Body Diagram of pilot (top and side view) showing net forces in the radial and transverse directions:

Equations of Motion:

\[ \sum F_r = m a_r = m \left( \dot{r} - r \dot{\theta}^2 \right) \]
\[ F_r = m \left[ -\frac{2\pi^2}{9} \cos \left( \frac{\pi}{3} t \right) - \left( 10 + 2 \cos \left( \frac{\pi}{3} t \right) \right) 0.1^2 \left( 4t - 1 \right)^2 \right] \]

\[ \sum F_\theta = m a_\theta = m \left( r \ddot{\theta} + 2r \dot{\theta} \right) \]
\[ F_\theta = m \left[ \left( 10 + 2 \cos \left( \frac{\pi}{3} t \right) \right) (0.4) - \frac{0.4\pi}{3} \sin \left( \frac{\pi}{3} t \right) (4t - 1) \right] \]

\[ \sum F_z = 0 \Rightarrow F_z = mg \]
PROBLEM 12.66 (Continued)

(a) Evaluate forces at \( t = 5 \) s:

\[
F_r = -221.78 \text{ lbs} \\
F_\theta = 61.37 \text{ lbs} \\
F_z = 175 \text{ lbs}
\]

\[
|F| = \sqrt{F_r^2 + F_\theta^2 + F_z^2} \\
|F| = 289.1 \text{ lb}
\]

(b) Plot of \( F_r \) and \( F_\theta \) from \( t = 0 \) to \( t = 10 \) s:
PROBLEM 12.70

Pin $B$ weighs 4 oz and is free to slide in a horizontal plane along the rotating arm $OC$ and along the circular slot $DE$ of radius $b = 20$ in. Neglecting friction and assuming that $\dot{\theta} = 15$ rad/s and $\ddot{\theta} = 250$ rad/s$^2$ for the position $\theta = 20^\circ$, determine for that position $(a)$ the radial and transverse components of the resultant force exerted on pin $B$, $(b)$ the forces $P$ and $Q$ exerted on pin $B$, respectively, by rod $OC$ and the wall of slot $DE$.

SOLUTION

Kinematics.

From the geometry of the system, we have

Then \[ r = 2b \cos \theta \quad \ddot{r} = -(2b \sin \theta) \ddot{\theta} \quad \dddot{r} = -2b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) \]
and \[ a_r = \dddot{r} - r \dddot{\theta} = -2b(\ddot{\theta} \sin \theta + \dot{\theta}^2 \cos \theta) - (2b \cos \theta) \dddot{\theta} = -2b(\ddot{\theta} \sin \theta + 2\dot{\theta}^2 \cos \theta) \]

Now \[ a_r = \dddot{r} - r \dddot{\theta} = -2 \left( \frac{20}{12} \right) [ (250 \text{ rad/s}^2) \sin 20^\circ + 2(15 \text{ rad/s}^2)^2 \cos 20^\circ ] = -1694.56 \text{ ft/s}^2 \]
and \[ a_\theta = \dot{r} \dddot{\theta} + 2r \dddot{\theta} = (2b \cos \theta) \dddot{\theta} + 2(-2b \dot{\theta} \sin \theta) \ddot{\theta} = 2b(\ddot{\theta} \cos \theta - 2\dot{\theta}^2 \sin \theta) \]
\[ = 2 \left( \frac{20}{12} \right) [ (250 \text{ rad/s}^2) \cos 20^\circ - 2(15 \text{ rad/s}^2)^2 \sin 20^\circ ] = 270.05 \text{ ft/s}^2 \]

Kinetics.

(a) We have \[ F_r = ma_r = \frac{\frac{1}{2} \text{ lb}}{32.2 \text{ ft/s}^2} \times (-1694.56 \text{ ft/s}^2) = -13.1565 \text{ lb} \]
and \[ F_\theta = ma_\theta = \frac{\frac{1}{2} \text{ lb}}{32.2 \text{ ft/s}^2} \times (270.05 \text{ ft/s}^2) = 2.0967 \text{ lb} \]

(b) \[ \Sigma F_r: \quad -F_r = -Q \cos 20^\circ \]
or \[ Q = \frac{1}{\cos 20^\circ} (13.1565 \text{ lb}) = 14.0009 \text{ lb} \]
\[ \Sigma F_\theta: \quad F_\theta = P - Q \sin 20^\circ \]
or \[ P = (2.0967 + 14.0009 \sin 20^\circ) \text{ lb} = 6.89 \text{ lb} \]

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PROBLEM 12.71

The two blocks are released from rest when $r = 0.8 \text{ m}$ and $\theta = 30^\circ$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block $A$ and the horizontal surface, determine (a) the initial tension in the cable, (b) the initial acceleration of block $A$, (c) the initial acceleration of block $B$.

SOLUTION

Let $r$ and $\theta$ be polar coordinates of block $A$ as shown, and let $y_B$ be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block $B$.

Constraint of cable: $r + y_B = \text{constant},$

\[ \dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \quad \text{or} \quad \ddot{r} = -a_B \quad (1) \]

For block $A$, $\Sigma F_x = m_A a_A$: $T \cos \theta = m_A a_A$ or $T = m_A a_A \sec \theta \quad (2)$

For block $B$, $\Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B \quad (3)$

Adding Eq. (1) to Eq. (2) to eliminate $T$, $m_B g = m_A a_A \sec \theta + m_B a_B \quad (4)$

Radial and transverse components of $\mathbf{a}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.

\[ \ddot{r} - r \dot{\theta}^2 = \mathbf{a}_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos \theta \quad (5) \]

Noting that initially $\dot{\theta} = 0$, using Eq. (1) to eliminate $\ddot{r}$, and changing signs gives

\[ a_B = a_A \cos \theta \quad (6) \]

Substituting Eq. (6) into Eq. (4) and solving for $a_A$,

\[ a_A = \frac{m_B g}{m_A \sec \theta + m_B \cos \theta} = \frac{(25)(9.81)}{20 \sec 30^\circ + 25 \cos 30^\circ} = 5.48 \text{ m/s}^2 \]

From Eq. (6), $a_B = 5.48 \cos 30^\circ = 4.75 \text{ m/s}^2$

(a) From Eq. (2), $T = (20)(5.48) \sec 30^\circ = 126.6 \quad T = 126.6 \text{ N} \quad \|$\n
(b) Acceleration of block $A$. $\mathbf{a}_A = 5.48 \text{ m/s}^2 \quad \|$\n
(c) Acceleration of block $B$. $\mathbf{a}_B = 4.75 \text{ m/s}^2 \quad \|$