PROBLEM 16.87

A 1.5-kg slender rod is welded to a 5-kg uniform disk as shown. The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10 rad/s clockwise, determine (a) the angular acceleration of the assembly, (b) the components of the reaction at C.

SOLUTION

Kinematics:

\[ \mathbf{a}_n = (CG)\alpha = (0.14 \text{ m})(10 \text{ rad/s}^2) \]
\[ \mathbf{a}_r = 14 \text{ m/s}^2 \]
\[ \mathbf{a}_e = (CG)\alpha = (0.14 \text{ m})\alpha \]

Kinetics:

\[ T_{\text{disk}} = \frac{1}{2} m_{\text{disk}} (CG)^2 \]
\[ = \frac{1}{2} (5 \text{ kg})(0.08 \text{ m})^2 \]
\[ = 16 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \]
\[ T_{\text{AB}} = \frac{1}{12} m_{\text{AB}} (AB)^2 \]
\[ = \frac{1}{12} (1.5 \text{ kg})(0.12 \text{ m})^2 \]
\[ = 1.8 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \]

(a) Angular acceleration.

\[ \Sigma M_C = \Sigma (M_C)_{\text{eff}}: \]
\[ W_{AB}(0.14 \text{ m}) = T_{\text{disk}}\alpha + m_{AB}\mathbf{a}_e(0.14 \text{ m}) + T_{AB}\alpha \]
\[ (1.5 \text{ kg})(9.81 \text{ m/s}^2)(0.14 \text{ m}) = T_{\text{disk}}\alpha + (1.5 \text{ kg})(0.14 \text{ m})^2\alpha + T_{AB}\alpha \]
\[ 2.060 \text{ N} \cdot \text{m} = (16 \times 10^{-3} + 29.4 \times 10^{-3} + 1.8 \times 10^{-3})\alpha \]
\[ 2.060 \text{ N} \cdot \text{m} = (47.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2)\alpha \]
\[ \alpha = 43.64 \text{ rad/s}^2 \]
\[ \alpha = 43.6 \text{ rad/s}^2 \]
PROBLEM 16.87 (Continued)

(b) Components of reaction of $C$.

$$\sum \Sigma F_x = \Sigma (F_x)_{\text{eff}} : \quad C_x = -m_{AB}\ddot{a}_x = -(1.5 \text{ kg})(14 \text{ m/s}^2)$$

$$C_x = -21.0 \text{ N} \quad C_x = 21.0 \text{ N} \quad \leftarrow \uparrow$$

$$\sum \Sigma F_y = \Sigma (F_y)_{\text{eff}} : \quad a_y = (0.14 \text{ m})(\alpha)$$

$$C_y - m_{\text{disk}}g - m_{AB}g = -m_{AB}\ddot{a}_y$$

$$C_y - (5 \text{ kg})9.81 - (1.5 \text{ kg})9.81 = -(1.5 \text{ kg})(0.14 \text{ m})(43.64 \text{ rad/s}^2)$$

$$C_y - 49.05 \text{ N} - 14.715 \text{ N} = -9.164 \text{ N}$$

$$C_y = +54.6 \text{ N} \quad \quad C_y = 54.6 \text{ N} \uparrow \downarrow$$
PROBLEM 16.88

Two identical 4-lb slender rods $AB$ and $BC$ are connected by a pin at $B$ and by the cord $AC$. The assembly rotates in a vertical plane under the combined effect of gravity and a 6 lb-ft couple $\mathbf{M}$ applied to rod $AB$. Knowing that in the position shown the angular velocity of the assembly is zero, determine $(a)$ the angular acceleration of the assembly, $(b)$ the tension in cord $AC$.

SOLUTION

$(a)$

\[ \Sigma M_A = (4 \text{ lb})(0.5 \text{ ft}) + (4 \text{ lb})(1.25 \text{ ft}) - 6 \text{ lb-ft} \]

\[ = 2 \left( \frac{1}{12} \right) \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (1 \text{ ft})^2 \alpha + \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (0.5 \text{ ft})^2 \alpha \]

\[ + \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right) (1.25 \text{ ft})^2 \alpha + \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right) [0.5 \text{ ft}(0.866)]^2 \alpha \]

\[ (7 - 6) \text{ lb-ft} = (0.26915) \alpha \]

\[ \alpha = 3.7154 \text{ rad/s}^2 \]

or \( \alpha = 3.72 \text{ rad/s}^2 \)

$(b)$

\[ \frac{4}{32.2} (1.25) \alpha \]

\[ \frac{4}{(32.2)^2} (0.5) (0.866) \alpha \]

\[ 0.5 \sin 33^\circ = 0.25 \text{ ft} \]

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PROBLEM 16.88 (Continued)

\[ \Sigma M_B = (4 \text{ lb})(0.25 \text{ ft}) - T(1 \text{ ft})(\sin 30^\circ) \]

\[ = \frac{1}{12} \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right)(1 \text{ ft})^2 \alpha + \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right)(1.25 \text{ ft})(0.25 \text{ ft}) \alpha \]

\[ + \left( \frac{4 \text{ lb}}{32.2 \text{ ft/s}^2} \right)(0.5 \text{ ft})(\cos 30^\circ)(0.433 \text{ ft}) \alpha \]

\[ = 0.07246 \alpha = (0.07246)(3.7154) = 1 - T(0.5) = 0.26922 \]

\[ T = 1.462 \text{ lb} \]

or \[ T = 1.462 \text{ lb} \]
**PROBLEM 16.122**

End $A$ of the 6-kg uniform rod $AB$ rests on the inclined surface, while end $B$ is attached to a collar of negligible mass which can slide along the vertical rod shown. When the rod is at rest a vertical force $P$ is applied at $B$, causing end $B$ of the rod to start moving upward with an acceleration of $4 \text{ m/s}^2$. Knowing that $\theta = 35^\circ$, determine the force $P$.

**SOLUTION**

**Kinematics:**

\[
a_B = 4 \text{ m/s}^2 \uparrow
\]

\[
(\omega = 0)
\]

\[
a_A = a_B + a_{AB}
\]

\[
a_d = 25^\circ = 4 \uparrow + 1.5 \alpha \searrow 35^\circ
\]

Law of sines:

\[
\frac{1.5 \alpha}{\sin 65^\circ} = \frac{4}{\sin 60^\circ}
\]

\[
\alpha = 2.7907 \text{ rad/s}^2
\]

\[
\bar{a} = a_G = a_B + a_{AB} = 4 \uparrow + 0.75 (2.7907) \searrow 35^\circ
\]

\[
\bar{a}_x = 0.75 (2.7907) \cos 35^\circ = 1.7145 \text{ m/s}^2 \rightarrow
\]

\[
\bar{a}_y = 4 \uparrow + 0.75 (2.7907) \sin 35^\circ = 2.7995 \text{ m/s}^2 \uparrow
\]

**Kinetics:**

\[
T = \frac{1}{12} ml^2 = \frac{1}{12} (6 \text{ kg})(1.5 \text{ m})^2 = 1.125 \text{ kg} \cdot \text{m}^2
\]

Law of sines:

\[
\frac{EB}{\sin 60^\circ} = \frac{AB}{\sin 65^\circ}, \quad EB = \frac{\sin 60^\circ (1.5)}{\sin 65^\circ} = 1.4333 \text{ m}
\]
PROBLEM 16.122 (Continued)

\[ ED = EB - DB = 1.4333 - 0.75 \cos 55^\circ = 1.0031 \text{ m} \]

\[ DG = 0.75 \sin 55^\circ = 0.61436 \text{ m} \]

\[ + \sum M = \sum (M_E)_{\text{eff}} : P(EB) - W(ED) = \bar{T\alpha} + m\bar{p}_x(\bar{DG}) + m\bar{p}_y(\bar{ED}) \]

\[ P(1.4333) - 6(9.81)(1.0031) = 1.125(2.7907) + (6)(1.7145)(0.61436) \]

\[ + (6)(2.7995)(1.0031) \]

\[ 1.4333P - 59.0425 = 3.1395 + 6.3199 + 16.8491 \]

\[ 1.4333P = 85.531 \]

\[ P = 59.5 \text{ N} \]