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Instantaneous Velocity

So we have found a way to assign a number to describe the motion over an interval of time, but we have not been able to describe the details of the motion.

We can improve the detail of our description of the motion by making the time interval shorter.

The extreme, or limit, of making the time interval shorter is called a *instant.*

The velocity over an arbitrarily short interval, or instant, is called the *instantaneous velocity.*

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{x}}{\Delta t}$$

The invention of the calculus supplied the mathematical rigor to the above definition, and the final solution to the problem of the measurement of motion.

Although we are not going to learn calculus in this course we can make a see what it does by examining the below graph of an object position versus time.

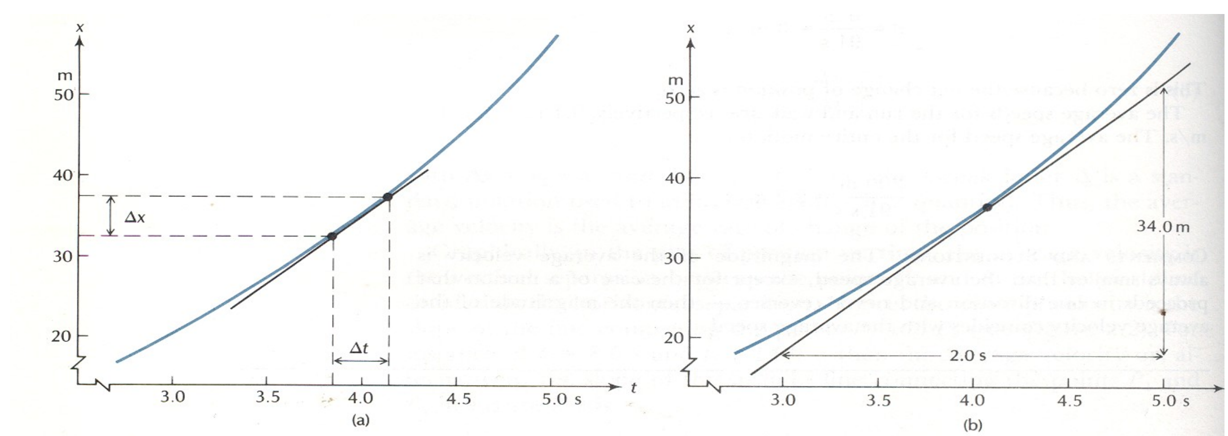
In graph (a.) we see that a line which connects 2 points on the position curve. The slope of this line is the rise over the run and it is

$$slope = \frac{\Delta \vec{x}}{\Delta t}$$

which is just our definition of the average velocity over the interval Δt

In the second graph we see that there is a line which is *tangent* to the curve, and therefore touches the curve at *only one point* . The progress from graph *a* to graph *b* is a graphical representation of *taking the limit as* Δt *goes to zero.* The 2 point which define the interval move closer and closer together till they become one.

The instantaneous velocity is the slope of the straight line which is tangent to the position vs time curve.



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Constant Velocity

If we have a make several measurements of the instantaneous velocity of an object over a time interval, and these measurement do not change then we say that the object is moving with a *constant velocity* over that time interval.

Speed and Velocity

Velocity is a vector which depends on position which is also a vector.

If we apply the same scheme for *distance*, which is a scalar, we get the concept of average and instantaneous *speed.*

Speed is the ratio of the change in distance over the change in time. If I calculate the *distance* between an object and some starting point s , then the speed will be given by

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

Note that there is no little arrow above the v indicating that it is a *scalar*.

The magnitude of the velocity vector is the speed.

$$v = |\vec{v}|$$

Changing velocity --- acceleration

Since we have defined instantaneous velocity we can now talk about the change in velocity.

We can form a mathematical object strictly analogous to the definition of velocity, the ration of the change in velocity to the time interval.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Furthermore we can define *instantaneous* acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

If we consider an analogy to the *position* vs *time* graph we conclude that

The acceleration is the slope of the tangent to the velocity vs. time graph.

Let's try these formulae with some examples.

Ex. A car is found to be traveling in the positive direction at a velocity of 10 miles per hour, at time = 5 seconds. At time = 9 seconds the car have come to a stop. Calculate the average acceleration of the car for the interval from 5 to 9 seconds.

To solve the problem we first write down the mathematical relationship we are going to use.

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$$\vec{a}_{av}=\frac{\vec{v}_2-\vec{v}_1}{t_2-t_1}$$

Now we write the variables in the above equation in terms of their values for this problem

$$\vec{v}_1=10\,miles/hr$$

$$\vec{v}_2=0\,miles/hr$$

$$t_1=5\,sec$$

$$t_2=9\,sec$$

Now we write the mathematical relationship with the quantities specific to out problem

$$\vec{a}_{av}=\frac{0\frac{miles}{hour}-10\frac{miles}{hour}}{9\,sec-5\,sec}$$

Now we notice in the above that we have 2 *different* units for the same quantity, time. This will not present a clear result, so we need to convert one of the time units to be the same as the other.

Let's say we want our result to be in terms of seconds so we can rewrite the above expression in the following manner.

$$\vec{a}_{av}=\frac{0\frac{miles}{hour}*\frac{1\,hour}{360\,seconds}-10\frac{miles}{hour}*\frac{1\,hour}{360\,seconds}}{9\,sec-5\,sec}$$

Note that I have multiplied each velocity by the number 1 in a special form. Multiplying by 1 leaves any quantity unchanged. I can no cancel the *hour* and get

$$\vec{a}_{av}=\frac{0\frac{miles}{sec}-.0027\frac{miles}{sec}}{4\,sec}$$

$$\vec{a}_{av}=-.000694\frac{miles}{second^2}$$

This answer is not yet acceptable form, we need to put it into scientific notation

$$\vec{a}_{av}=-6.94*10^{-4}\frac{miles}{sec^2}$$

Let's look at the this answer. First it has a negative sign, and we know that the the car is slowing down to a stop, so for a car with *positive* initial velocity, a negative acceleration means that it is slowing down.

To understand the what the dimensions are telling us let's write the answer as

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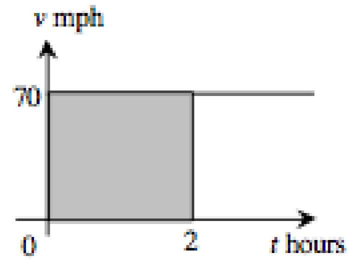
$$\vec{a}_{av} = -6.94 * 10^{-4} \frac{\frac{miles}{sec}}{sec}$$

This means that the velocity changes by $-6.94 * 10^{-4} \frac{miles}{sec}$ every second.

A special case: Constant Acceleration

Velocity-time graphs

Constant velocity
The sketch shows the velocity-time graph for a car that is travelling along a motorway at a steady 70 mph.



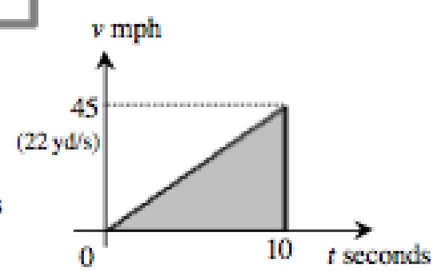
The area under this graph is rectangular in shape.

The shaded area = 2 × 70 = 140

This is the distance in miles travelled in 2 hours when the speed is 70mph.

Area under a velocity-time graph = distance travelled.

Constant acceleration
Now consider the case when a car is accelerating steadily from 0 to 45 mph in 10 seconds.



The graph shows this situation, but note that the velocity is in miles per hour whilst the time is in seconds. The units need to be converted in order to find the distance travelled.

$$45 \text{ mph} = \frac{45 \times 1760}{60 \times 60} = 22 \text{ yards per second.}$$

(1 mile = 1760 yards)

The distance travelled is given by the area under the graph:

$$\text{Distance travelled} = \frac{10 \times 22}{2} = 110 \text{ yards.}$$

Note that this is equivalent to the car travelling for 10 seconds at the average speed of 11 yards per second.

In each case sketch a velocity-time graph and find the distance travelled in miles or yards:

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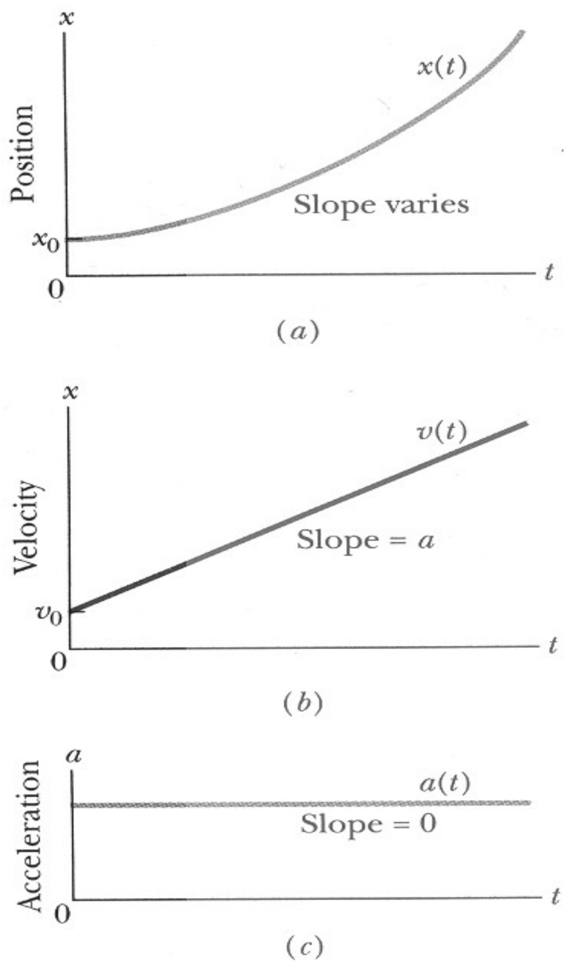

If we look at the above graphs of an object undergoing constant acceleration we see the following relationships.

Slope

1. The acceleration is a constant and is equal to the slope of the velocity vs time graph.
2. The velocity is increasing and it gives the increasing slope of the tangent to the position vs time graph.

Area

1. The area under the acceleration graph, over some time interval, is equal to the change in the velocity over that same time interval
2. The area under the velocity vs. time graph, over some time interval, is equal to the change in the position of the object over that same time interval.

The above statements are consequences of the FUNDAMENTAL THEOREM OF CALCULUS and provide the solution to the problem of the measurement of motion first stated by the Greeks.

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Equations of Constant Acceleration.

If we state algebraically the above conclusions we get the following 4 equations for an object which starts out at moving at a velocity v_0 at time $t = 0$, and accelerates at a constant acceleration a

$$\vec{v} = \vec{v}_0 + \vec{a}\,t \qquad \text{(2-11a)}$$

$$\vec{x} = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2 \qquad \text{(2-11b)}$$

$$v^2 = {v_0}^2 + 2a(x - x_0) \qquad \text{(2-11c)}$$

In addition we have, for cases of constant acceleration

$$\vec{v}_{av} = \frac{\vec{v} + \vec{v}_0}{2} \qquad \text{(2-11d)}$$

We see from Equation 2-11b that

For an object moving with constant acceleration the position is proportional to the SQUARE OF THE TIME

Galileo's Experiment

Galileo determined that object which we rolling down a smooth ramp, under the influence of gravity, the distance was precisely proportional to the elapsed time SQUARE.

The reverse inference is also valid, object whose displacement is proportional to the square of the time are undergoing *constant acceleration.*

Homework

Ch. 2. Problem #6, #10 , #17, #25 (Use Student Solution manual to find similar worked out problems)

CLICKER QUIZ

A car slows down from 23 m/s to rest in a distance of 85 m. What is its acceleration?