Exam 1 Solution

1. Note that it will be easier to work with $P(\overline{A}) = P(\overline{A} \cap \overline{B}) + P(\overline{A} \cap B)$, since we have $P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$ and $P(\overline{A} \cap B) = 1 - P(A \cup \overline{B}) = 1 - 0.9 = 0.1$ (by DeMorgan's laws), so that $P(\overline{A}) = P(\overline{A} \cap \overline{B}) + P(\overline{A} \cap B) = 0.3 + 0.1 = 0.4$. Hence

$$P(A) = 1 - P(\overline{A}) = 1 - 0.4 = 0.6$$

2. Let $M = \{\text{Man}\}$ and $\overline{M} = W = \{\text{Woman}\}$. Let $Sm = \{\text{Smoker}\}$. Then we have that

$$P(M) = 4/10, \quad P(\overline{M}) = P(W) = 6/10, \quad P(Sm|M) = 1/4, \quad P(Sm|\overline{M}) = 1/5$$

   (a) Using the law of total probability,

$$\begin{aligned} P(Sm) &= P(\{Sm \cap M\} \cup \{Sm \cap \overline{M}\}) = P(Sm \cap M) + P(Sm \cap \overline{M}) \\ &= P(Sm|M)P(M) + P(Sm|\overline{M})P(\overline{M}) = (1/4)(4/10) + (1/5)(6/10) = 11/50 \end{aligned}$$

   (b) Using part (a) or Bayes Rule,

$$P(M|Sm) = \frac{P(M \cap Sm)}{P(Sm)} = \frac{P(Sm|M)P(M)}{P(Sm)} = \frac{(1/4)(4/10)}{11/50} = \frac{5}{11}$$

3. Note that the Y take values $1, 2, \ldots, 6$. Then

$$\begin{aligned} P(Y = 1) &= \frac{1}{6} \\ P(Y = 2) &= \frac{5}{6}\frac{1}{5} = \frac{1}{6} \\ P(Y = 3) &= \frac{5}{6}\frac{4}{5}\frac{1}{4} = \frac{1}{6} \\ &\vdots \\ P(Y = 6) &= \frac{5}{6}\frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2}\frac{1}{1} = \frac{1}{6} \end{aligned}$$

   Therefore,

$$P(Y = y) = \frac{1}{6}$$

   for all $y = 1, \ldots, 6$.