Homework 1

Due Thursday, January 25

1. If A and B are events, show the following by (i) drawing the Venn diagram and (ii) using the identities $A = A \cap S$ and $S = B \cup \overline{B}$.
   (a) $A = (A \cap B) \cup (A \cap \overline{B})$
   (b) If $B \subset A$, then $A = B \cup (A \cap \overline{B})$
2. If A and B are events, show that
   (a) $P(A) = P(A \cap B) + P(A \cap \overline{B})$
   (b) $P(A \cap B) = P(B) - P(\overline{A} \cap B)$
   (c) If $B \subset A$, is it true that $P(A) = P(B) + P(A \cap \overline{B})$? Please explain why or why not.
3. Suppose that A and B are two events. Write expressions involving unions, intersections, and complements that describe the following:
   (a) Both events occur.
   (b) At least one occurs.
   (c) Neither occurs.
   (d) Exactly one occurs.
4. Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2% have defects in both shafts and bushings. One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has
   (a) a bushing defect?
   (b) a shaft or bushing defect?
   (c) exactly one of the two types of defects?
   (d) neither type of defect?
5. According to a survey, 70% of respondents have health insurance, 30% have disability income insurance, and 60% have only health insurance. What is the probability that a randomly selected respondent has only disability income insurance?
6. Suppose two balanced coins are tossed and the upper faces are observed.
   (a) List the sample points for this experiment.
   (b) Assign a reasonable probability to each sample point. (Are the sample points equally likely?)
   (c) Let A denote the event that exactly one head is observed and B the event that at least one head is observed. List the sample points in both A and B.
   (d) From your answer to part (c), find $P(A)$, $P(B)$, $P(A \cap B)$, $P(A \cup B)$, and $P(\overline{A} \cup B)$

7. A business office orders paper supplies from one of three vendors, V_1, V_2, or V_3. Orders are to be placed on two successive days, one order per day. Thus, (V_2, V_3) might denote that vendor V_2 gets the order on the first day and vendor V_3 gets the order on the second day (the same vendor can be repeated the next day).

   (a) List the sample points in this experiment of ordering paper on two successive days.

   (b) Assume the vendors are selected at random each day and assign a probability to each sample point.

   (c) Let A denote the event that the same vendor gets both orders and B the event that V_2 gets at least one order. Find $P(A)$, $P(B)$, $P(A \cup B)$, and $P(A \cap B)$.

8. Four equally qualified people apply for two identical positions in a company. One and only one applicant is a member of a minority group. The positions are filled by choosing two of the applicants at random.

   (a) List the possible outcomes for this experiment.

   (b) Assign reasonable probabilities to the sample points.

   (c) Find the probability that the applicant from the minority group is selected for a position.

9. **(5090*)** If the events $A_1, A_2, \ldots$ form a partition of S, and B is another event, show that

$$P(B) = \sum_{i=1}^{\infty} P(B \cap A_i)$$