Homework 1

Due Thursday, January 30

NOTE: *The questions marked with* **(5090*)** *are required for 5090 students, optional for 4070 students.*

1. How many different seven-digit telephone numbers can be formed if the first digit cannot be zero?
2. How many different letter arrangements can be made from the letters
   (a) Fluke?
   (b) Propose?
   (c) Mississippi?
3. In how many ways can 8 people be seated in a row if
   (a) there are no restrictions on the seating arrangement?
   (b) there are 5 men and they must sit next to each other?
   (c) there are 4 married couples and each couple must sit together?
4. A student has to sell 2 books from a collection of 6 math, 7 science, and 4 economics books. How many choices are possible if both books are to be on the same subject? How about if the books are to be on different subjects?
5. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
   (a) 2 of the men refuse to serve together?
   (b) 2 of the women refuse to serve together?
   (c) 1 man and 1 woman refuse to serve together?
6. The game of bridge is played by 4 players, each of whom is dealt 13 cards. How many bridge deals are possible?
7. Ten weight lifters are competing in a team weight lifting contest. Of the lifters, 3 are from the United States, 4 are from Russia, 2 are from China, and 1 is from Canada. If the scoring takes account of the countries that the lifters represent, but not their individual identities, how many different outcomes are possible from the point of view of scores? How many different outcomes correspond to results in which the United States has 1 competitor in the top three and 2 in the bottom three?
8. Expand $(3x^2 + y)^4$.

9. Show that

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

10. Show that, for $n > 0$,

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

11. Show that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

(HINT: You may find

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$$

useful)

12. Show, using only the definition $\binom{m}{k} = \frac{m!}{k!(m-k)!}$, that

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}, \quad 1 \leq r \leq n$$

13. **(5090*)** Show that

$$\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$$

14. **(5090*)** If $0 < p < 1$, show that

$$\sum_{k=1}^{\infty} kp^{k-1} = \frac{1}{(1-p)^2}$$

(HINT: $\sum_{k=0}^{\infty} p^k =?$).

15. **(5090*)** From 27 pieces of luggage, an airline luggage handler damages a random sample of four. Let x be the number (unknown) of insured luggage out of the 27 luggage.
    (a) Write down the event that exactly two of the four damaged luggages are insured.
    (b) Find x if the number that exactly one of the damaged pieces of luggage is insured is twice the number that none of the damaged pieces are insured.