Homework 2 Due Thursday, February 6

NOTE: The questions marked with (5090*) are required for 5090 students, optional for 4070 students.

- 1. Suppose that the events E_1 , E_2 , E_3 , E_4 , E_5 partition a sample space S.
 - (a) If $P(E_1) = P(E_2) = 0.15$, $P(E_3) = 0.4$, and $P(E_4) = 2P(E_5)$, find the probabilities of E_4 and E_5 .
 - (b) If $P(E_1) = 3P(E_2) = 0.3$, find the probabilities of the remaining events if you know that the remaining events are equally probable.
- 2. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.
 - (a) What percentage of males smokes neither cigars nor cigarettes?
 - (b) What percentage smokes cigars but not cigarettes?
- 3. Hydraulic landing assemblies coming from an aircraft rework facility are each inspected for defects. Historical records indicate that 8% have defects in shafts only, 6% have defects in bushings only, and 2% have defects in both shafts and bushings. One of the hydraulic assemblies is selected randomly. What is the probability that the assembly has
 - (a) a bushing defect?
 - (b) a shaft or bushing defect?
 - (c) exactly one of the two types of defects?
 - (d) neither type of defect?
- 4. According to a survey, 70% of respondents have health insurance, 30% have disability income insurance, and 60% have only health insurance. What is the probability that a randomly selected respondent has only disability income insurance?
- 5. A hospital administrator codes incoming patients suffering gunshot wounds according to whether they have insurance (coding 1 if they do and 0 if they do not) and according to their condition, which is rated as good (g), fair (f), or serious (s). Consider an experiment that consists of the coding of such a patient.
 - (a) Give the sample space of this experiment.
 - (b) Let A be the event that the patient is in serious condition. Specify the outcomes in A.
 - (c) Let B be the event that the patient is uninsured. Specify the outcomes in B.
 - (d) What is the outcome $A \cap B$?
 - (e) Give all the outcomes in the event $B^c \cup A$.

- 6. Let E, F, and G be three events. Find expressions for the events so that, of E, F, and G,
 - (a) only E occurs
 - (b) at least two of the events occur
 - (c) all three events occur
 - (d) none of the events occurs
 - (e) at most two of the events occur
 - (f) exactly two of the events occur
- 7. A student prepares for an exam by studying a list of ten problems. She can solve six of them. For the exam, the instructor selects five problems at random from the ten on the list given to the students. What is the probability that the student can solve all five problems on the exam?
- 8. What is the probability of being dealt a poker hand that is
 - (a) a flush?(A hand is said to be a flush if all 5 cards are of the same suit.)
 - (b) one pair? (This occurs when the cards have denominations *a*, *a*, *b*, *c*, *d*, where *a*, *b*, *c*, and *d* are all distinct.)
 - (c) three of a kind? (This occurs when the cards have denominations a, a, a, b, c, where a, b, and c are all distinct.)
- 9. Suppose that you have 4 white socks, 2 black socks, 6 red socks, and 3 green socks. Assume that all socks are indistinguishable otherwise. Suppose that you draw 4 socks, all at once.
 - (a) What is the probability that you get 2 socks of the same color and the remaining 2 socks are of the same color, but not all 4 socks are of the same color?
 - (b) What is the probability that you draw at least one red sock.
- 10. Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue, and 18 green balls. Find the probability that
 - (a) 3 red, 2 blue, and 2 green balls are withdrawn;
 - (b) at least 2 red balls are withdrawn;
 - (c) all withdrawn balls are the same color;
 - (d) either exactly 3 red balls or exactly 3 blue balls are withdrawn.
- 11. An urn contains 3 red and 7 black balls. Players A and B withdraw balls from the urn consecutively until a red ball is selected. Find the probability that A selects the red ball. (A draws the first ball, then B, and so on. There is no replacement of the balls drawn.)
- 12. A woman has n keys, of which one will open her door. If she tries the keys at random, discarding those that do not work, what is the probability that she will open the door on her *k*th try? What if she does not discard previously tried keys?

- 13. A group of individuals containing b boys and g girls is lined up in random order; that is, each of the (b+g)! permutations is assumed to be equally likely. What is the probability that the person in the *i*th position is a girl?
- 14. A manufacturer has nine distinct motors in stock, two of which came from a particular supplier. The motors must be divided among three production lines, with three motors going to each line. If the assignment of motors to lines is random, find the probability that both motors from the particular supplier are assigned to the first line.
- 15. Show that

$$P(EF^c) = P(E) - P(EF).$$

16. (5090*) Prove Bonferroni's inequality

$$P(EF) \ge P(E) + P(F) - 1$$

then use induction to generalize Bonferroni's inequality to n events

$$P(E_1E_2\cdots E_n) \ge P(E_1) + P(E_2) + \cdots + P(E_n) - (n-1)$$

- 17. (5090*) A construction worker experiences up to five accidents this year. The probability of no accidents is three times the probability of exactly one accident. The probability of exactly one accident is four times the probability of exactly two accidents. The probability of exactly two accidents is five times the probability of exactly three accidents. The probability of three or fewer accidents is 0.95. Calculate the probability of two or fewer accidents.
- 18. (5090*) From 27 pieces of luggage, an airline luggage handler damages a random sample of four. The probability that exactly one of the damaged pieces of luggage is insured is twice the probability that none of the damaged pieces are insured. Calculate the probability that exactly two of the four damaged pieces are insured.