

Homework 2

Due Thursday, February 1

1. How many different seven-digit telephone numbers can be formed if the first digit cannot be zero?
2. In how many ways can 8 people be seated in a row if
 - (a) there are no restrictions on the seating arrangement?
 - (b) there are 5 men and they must sit next to each other?
 - (c) there are 4 married couples and each couple must sit together?
3. A student has to sell 2 books from a collection of 6 math, 7 science, and 4 economics books. How many choices are possible if both books are to be on the same subject? How about if the books are to be on different subjects?
4. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
 - (a) 2 of the men refuse to serve together?
 - (b) 2 of the women refuse to serve together?
 - (c) 1 man and 1 woman refuse to serve together?
5. A balanced die is tossed six times, and the number on the uppermost face is recorded each time. What is the probability that the numbers recorded are 1, 2, 3, 4, 5, and 6 in any order?
6. A student prepares for an exam by studying a list of ten problems. She can solve six of them. For the exam, the instructor selects five problems at random from the ten on the list given to the students. What is the probability that the student can solve all five problems on the exam?
7. Five cards are dealt from a standard 52-card deck. What is the probability that we draw
 - (a) 3 aces and 2 kings?
 - (b) a “full house” (3 cards of one kind, 2 cards of another kind)?
8. How many different letter arrangements can be made from the letters
 - (a) Fluke?
 - (b) Propose?
 - (c) Mississippi?

9. Ten weight lifters are competing in a team weight lifting contest. Of the lifters, 3 are from the United States, 4 are from Russia, 2 are from China, and 1 is from Canada. If the scoring takes account of the countries that the lifters represent, but not their individual identities, how many different outcomes are possible from the point of view of scores? How many different outcomes correspond to results in which the United States has 1 competitor in the top three and 2 in the bottom three?
10. A manufacturer has nine distinct motors in stock, two of which came from a particular supplier. The motors must be divided among three production lines, with three motors going to each line. If the assignment of motors to lines is random, find the probability that both motors from the particular supplier are assigned to the first line.

11. Show that

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

(HINT: Use the binomial theorem).

12. Show that, for $n > 0$,

$$\sum_{k=0}^n (-1)^k \binom{n}{k} = 0$$

13. (5090*) Show that

$$\sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$

14. (5090*) If $0 < p < 1$, show that

$$\sum_{k=1}^{\infty} kp^{k-1} = \frac{1}{(1-p)^2}$$

(HINT: $\sum_{k=0}^{\infty} p^k = ?$).

15. (5090*) From 27 pieces of luggage, an airline luggage handler damages a random sample of four. The probability that exactly one of the damaged pieces of luggage is insured is twice the probability that none of the damaged pieces are insured. Calculate the probability that exactly two of the four damaged pieces are insured.