Homework 3
Due Thursday, February 8

1. If two events, A and B, are such that $P(A) = 0.5$, $P(B) = 0.3$, and $P(A \cap B) = 0.1$, find the following:

   (a) $P(A|B)$

   (b) $P(B|A)$

   (c) $P(A|A \cup B)$

   (d) $P(A|A \cap B)$

   (e) $P(A \cap B|A \cup B)$

2. Two events A and B are such that $P(A) = 0.2$, $P(B) = 0.3$, and $P(A \cup B) = 0.4$. Find the following:

   (a) $P(A \cap B)$

   (b) $P(\overline{A} \cup \overline{B})$

   (c) $P(\overline{A} \cap \overline{B})$

   (d) $P(\overline{A}|B)$

3. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the probability that she passes the second one is 0.8, and if she passes both the first and the second exams, then the probability that she passes the third exam is 0.7. What is the probability that she passes all three exams?

4. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i? Compute for all values of i between 2 and 12. (Hint: some values should be 0.)

5. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a sum of 5 occurs before a sum of 7.

6. A woman has n keys, of which one will open her door. If she tries the keys at random, discarding those that do not work, what is the probability that she will open the door on her kth try? What if she does not discard previously tried keys?

7. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

8. People in groups A and B are observed to react differently to a given set of circumstances. It has been observed that 70% of those in B group react positively to these circumstances, whereas only 40% of those in A group react positively. A total 20 people (5 in group A, 15 in group B) was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire. A response picked at random from the 20 was negative. What is the probability that it was that from group A?

9. A total of 46 percent of the voters in a certain city classify themselves as Independents, whereas 30 percent classify themselves as Liberals and 24 percent say that they are Conservatives. In a recent local election, 35 percent of the Independents, 62 percent of the Liberals, and 58 percent of the Conservatives voted. A voter is chosen at random. Given that this person voted in the local election, what is the probability that he or she is (a) an Independent? (b) a Liberal? (c) a Conservative? Also, (d) What fraction of voters participated in the local election?

10. Consider two boxes, one containing 1 black and 1 white marble, the other 2 black and 1 white marble. A box is selected at random, and a marble is drawn from it at random. What is the probability that the marble is black? What is the probability that the first box was the one selected given that the marble is white?

11. A box contains 5 white and 10 black marbles. A fair die is rolled and that number of marbles is randomly chosen from the box. What is the probability that all of the marbles selected are white? What is the conditional probability that the die landed on 3 if all the marbles selected are white?

12. If A and B are mutually exclusive events, with $P(B) > 0$. show that

$$P(A|A \cup B) = \frac{P(A)}{P(A) + P(B)}$$

13. Prove that $P(A \cap B) \geq 1 - P(\overline{A}) - P(\overline{B})$.

14. **(5090*)** An accident victim will die unless in the next 10 minutes he receives some type A, Rh+ blood, which can be supplied by a single donor. The hospital requires 2 minutes to type a prospective donor's blood and 2 minutes to complete the transfer of blood. Many untyped donors are available, and 40% of them have type A, Rh+ blood. What is the probability that the accident victim will be saved if only one blood-typing kit is available? Assume that the typing kit is reusable but can process only one donor at a time.

15. **(5090*)** Players A and B alternate flipping a coin, and the first player to obtain a head wins. Assume that A flips the coin first.

   (a) Suppose that $P(H) = p$. Find the probability that A wins.
   (b) Show that $P(A \text{ wins}) > 0.5$.