Homework 4

Due Thursday, February 20

- 1. Suppose that a die is rolled twice. What are the possible values that the following random variables can take on:
 - (a) the maximum value to appear in the two rolls;
 - (b) the minimum value to appear in the two rolls;
 - (c) the sum of the two rolls;
 - (d) the value of the first roll minus the value of the second roll?
 - (e) If the die is assumed fair, calculate the probabilities associated with the random variables in part (d).

(For parts (a) to (d), only list the possible values for the random variables, not probabilities. Also, assume that ties, such as (1, 1) or (6, 6), are allowed.)

- 2. Four independent flips of a fair coin are made. Let X denote the number of heads obtained. Let Y = X - 2.
 - (a) What are the possible values of Y.
 - (b) Calculate and plot the probability mass function of the random variable Y.
- 3. The number of injury claims per month is modeled by a random variable Y with

$$P(X = x) = \frac{1}{(x+1)(x+2)}, \ x = 0, 1, 2, \dots$$

Calculate the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

- 4. An urn contains four chips numbered 1 through 4. Two are drawn without replacement. Let the random variable X denote the larger of the two (no ties). Find E[X].
- 5. A gambling book recommends the following "winning strategy" for the game of roulette: Bet 1 on red. If red appears (which has probability 18/38), then take the 1 profit and quit. If red does not appear and you lose this bet (which has probability 20/38 of occurring), make additional 1 bets on red on each of the next two spins of the roulette wheel and then quit. We assume that each spin is independent (does not depend on the previous spin result by, say, spinning it several rounds before spinning for the result). Let X denote your winnings when you quit.
 - (a) List all possible values of X.
 - (b) Find P(X > 0).
 - (c) Find E[X].

- 6. Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus.
 - (a) List all possible values of X and Y. Show that X and Y take on the same values but that $P(X = i) \neq P(Y = i)$. (Hint: Note how the selection mechanism differs between X and Y.)
 - (b) Find E[X] and E[Y].
 - (c) Find Var(X) and Var(Y).
- 7. If E[X] = 1 and Var(X) = 5, find
 - (a) $E[(2+X)^2]$.
 - (b) Var(4+3X).
- 8. Auto accidents for an individual driver can result in annual losses of 0, 1000, 5000, 10000, or 15000 with probabilities 0.75, 0.12, 0.08, 0.04, and 0.01, respectively. An auto insurer offers a policy that insures individual drivers against such losses, subject to an annual deductible of 500. The insurer charges an annual premium that exceeds its expected annual payment by 75 to provide for insurer expenses and profit. Calculate the annual premium that the insurer charges. (HINT: The "annual payment" in this case is given by Y, which is defined, along with "loss/claim" X and "deductible" d, in the Insurance Deductible section on page 3 of Exam P Formula Sheet; see the class webpage, may also consult "Risk and Insurance" for more detail).
- 9. A ball is drawn from an urn containing 3 white and 3 black balls. After the ball is drawn, it is replaced and another ball is drawn. This process goes on indefinitely. What is the probability that, of the first 4 balls drawn, exactly 2 are white?
- 10. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding (i.e., three or more digit 'wins'; e.g., 00101=0, 11011=1, etc.), what is the probability that the message will be wrong when decoded? What independence assumptions are you making?
- 11. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?
- 12. A company prices its hurricane insurance using the following assumptions: (i) In any calendar year, there can be at most one hurricane. (ii) In any calendar year, the probability of a hurricane is 0.05. (iii) The numbers of hurricanes in different calendar years are mutually independent. Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

- 13. A student is getting ready to take an important oral examination and is concerned about the possibility of having an "on" day or an "off" day. He figures that if he has an on day, then each of his examiners will pass him, independently of each other, with probability 0.8, whereas if he has an off day, this probability will be reduced to 0.4. Suppose that the student will pass the examination if a majority of the examiners pass him. If the student feels that he is twice as likely to have an off day as he is to have an on day, should he request an examination with 3 examiners or with 5 examiners?
- 14. Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are H, T, T (meaning that the first flip results in heads, the second in tails, and the third in tails).
- 15. A manufacturer produces computers and releases them in shipments of 100. Out of 100, the probability that exactly three computers are defective is twice the probability that exactly two computers are defective. The events that different computers are defective are mutually independent. Calculate the probability that a randomly selected computer is defective.
- 16. A particular sale involves four items randomly selected from a large lot that is known to contain 10% defectives. Let X denote the number of defectives among the four sold. The purchaser of the items will return the defectives for repair, and the repair cost is given by $C = 3X^2 + X + 2$. Find the expected repair cost.
- 17. (5090*) If $X \sim \text{Binomial}(n, p)$ show that

$$P(X > 1 | X \ge 1) = \frac{1 - (1 - p)^n - n(1 - p)^{n-1}p}{1 - (1 - p)^n}$$

18. (5090*) An actuary has done an analysis of all policies that cover two cars. 70% of the policies are of type A for both cars, and 30% of the policies are of type B for both cars. The number of claims on different cars across all policies are mutually independent. The distributions of the number of claims on a car are given in the following table.

#	of	Claims	Туре А	Туре В
0			40%	25%
1			30%	25%
2			20%	25%
3			10%	25%

Four policies are selected at random. Calculate the probability that exactly one of the four policies has the same number of claims on both covered cars.

19. (5090^{*}) If there are n trials in a binomial experiment and we observe x successes, both of which are fixed, find the value of p that maximizes

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

20. (5090*) If X is a discrete random variable taking on values $0, 1, 2, \ldots$, show that

$$E[X] = \sum_{k=0}^{\infty} P(X > k)$$