Homework 4
Due Thursday, February 15

1. A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for Y, the number of correct matches.

2. Suppose that a die is rolled twice. What are the possible values that the following random variables can take on:

   (a) the sum of the two rolls;

   (b) the value of the first roll minus the value of the second roll;

   (c) If the die is assumed fair, calculate the probabilities associated with the random variables in part (b).

   (For parts (a) and (b), just list the possible values for the random variables, not probabilities.)

3. The number of injury claims per month is modeled by a random variable Y with

$$P(Y = y) = \frac{1}{(y+1)(y+2)},\ y = 0, 1, 2, \ldots$$

   Calculate the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

4. In a gambling game a person draws a single card from an ordinary 52-card playing deck. A person is paid \$15 for drawing a jack or a queen and \$5 for drawing a king or an ace. A person who draws any other card pays (loses) \$4. If a person plays this game, what is the expected gain?

5. A gambling book recommends the following "winning strategy" for the game of roulette: Bet \$1 on red. If red appears (which has probability 18/38), then take the \$1 profit and quit. If red does not appear and you lose this bet (which has probability 20/38 of occurring), make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. We assume that each spin is independent. Let Y denote your winnings when you quit.

   (a) List all possible values of Y.

   (b) Find $P(Y > 0)$.

   (c) Find $E(Y)$.

6. An urn contains four chips numbered 1 through 4. Two are drawn without replacement. Let the random variable Y denote the larger of the two. Find $E(Y)$.

7. If $E(Y) = 1$ and $\text{Var}(Y) = 5$, find (a) $E((2 + Y)^2)$, and (b) $\text{Var}(4 + 3Y)$.

8. The probability that a patient recovers from a stomach disease is 0.7. Suppose 10 people are known to have contracted this disease. What is the probability that

   (a) exactly 4 recover?
   (b) at least 3 recover?
   (c) at least 5 but not more than 7 recover?
   (d) at most 8 recover?

9. A communications channel transmits the digits 0 and 1. However, due to static, the digit transmitted is incorrectly received with probability 0.2. Suppose that we want to transmit an important message consisting of one binary digit. To reduce the chance of error, we transmit 00000 instead of 0 and 11111 instead of 1. If the receiver of the message uses "majority" decoding (i.e., three or more digit 'wins'; e.g., 00101=0, 11011=1, etc.), what is the probability that the message will be wrong when decoded? What independence assumptions are you making?

10. A complex electronic system is built with a certain number of backup components in its subsystems. One subsystem has four identical components, each with a probability of 0.2 of failing in less than 1000 hours. The subsystem will operate if any two of the four components are operating. Assume that the components operate independently. Find the probability that

    (a) exactly two of the four components last longer than 1000 hours.
    (b) the subsystem operates longer than 1000 hours.

11. On a multiple-choice exam with 3 possible answers for each of the 5 questions, what is the probability that a student will get 4 or more correct answers just by guessing?

12. A student is getting ready to take an important oral examination and is concerned about the possibility of having an "on" day or an "off" day. He figures that if he has an on day, then each of his examiners will pass him, independently of each other, with probability 0.8, whereas if he has an off day, this probability will be reduced to 0.4. Suppose that the student will pass the examination if a majority of the examiners pass him. If the student feels that he is twice as likely to have an off day as he is to have an on day, should he request an examination with 3 examiners or with 5 examiners?

13. **(5090*)** Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are H, T, T (meaning that the first flip results in heads, the second in tails, and the third in tails).

14. **(5090*)** If $Y \sim \text{Binomial}(n, p)$ show that

$$P(Y > 1 | Y \geq 1) = \frac{1 - (1-p)^n - n(1-p)^{n-1}p}{1 - (1-p)^n}$$

15. **(5090*)** If Y is a discrete random variable taking on values $0, 1, 2, \ldots$, show that

$$E(Y) = \sum_{k=0}^{\infty} P(Y > k)$$