Homework 5

Due Thursday, February 22

1. Consider a roulette wheel consisting of 38 numbers: 1 through 36, 0, and double 0 (00). If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that

   (a) Smith will lose his first 5 bets?

   (b) his first win will occur on his fourth bet?

2. About six months into George W. Bush's second term as president, a Gallup poll indicated that a near record (low) level of 41% of adults expressed "a great deal" or "quite a lot" of confidence in the U.S. Supreme Court. Suppose that you conducted your own telephone survey at that time and randomly called people and asked them to describe their level of confidence in the Supreme Court. Find the probability distribution for Y, the number of calls until the first person is found who does not express "a great deal" or "quite a lot" of confidence in the U.S. Supreme Court.

3. If Y is a geometric random variable with success probability p,

   (a) show that $P(Y > a) = (1 - p)^a$.

   (b) using the previous part, show that $P(Y = a + k | Y > a) = P(Y = k)$.

4. Suppose that a batch of 100 items contains 6 that are defective and 94 that are not defective. If Y is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) $P(Y = 0)$ and (b) $P(Y > 2)$.

5. A purchaser of transistors buys them in a lot of 20. It is his policy to randomly inspect 4 components from a lot and to accept the lot only if all 4 are nondefective. What proportion of lots does the purchaser reject

   (a) if a lot has 2 defective components?

   (b) if each component in a lot is, independently, defective with probability 0.1?

6. A jury of 6 persons was selected from a group of 20 potential jurors, of whom 8 were African American and 12 were white. The jury was supposedly randomly selected, but it contained only 1 African American member. Do you have any reason to doubt the randomness of the selection?

7. Twenty identical looking packets of white powder are such that 15 contain cocaine and 5 do not. Four packets were randomly selected, and the contents were tested and found to contain cocaine. Two additional packets were selected from the remainder and sold by undercover police officers to a single buyer. What is the probability that the 6 packets randomly selected are such that the first 4 all contain cocaine and the 2 sold to the buyer do not?

8. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be

   (a) at least 2 such accidents in the next month?

   (b) at most 1 accident in the next month?

   Assume Poisson distribution.

9. Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$.

   (a) Find the probability that 3 or more accidents occur today.

   (b) Find the probability that 3 or more accidents occur today given that at least 1 accident occurred today already.

10. A parking lot has two entrances. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour and at entrance II according to a Poisson distribution at an average of four per hour. What is the probability that a total of three cars will arrive at the parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent.)

11. Consider a binomial experiment for $n = 20$, $p = 0.05$. Calculate the binomial probabilities for $Y = 0, 1, 2$. Calculate the same probabilities by using the Poisson approximation with $\lambda = np$. Compare.

12. **(5090*)** If Y has a geometric distribution with success probability p, what is the probability of Y being an odd integer?

13. **(5090*)** If Y has a geometric distribution with success probability p, find $\text{Var}(Y)$.

    (Hint: First compute $E(Y(Y-1))$ by using the proof techniques in Theorem 3.8).

14. **(5090*)** If Y is a Poisson random variable with rate λ, show that

$$P(Y = y) = \frac{\lambda}{y} P(Y = y - 1)$$

15. **(5090*)** The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send three employees who have positive indications of asbestos on to a medical center for further testing. If 40% of the employees have positive indications of asbestos in their lungs, find the probability that ten employees must be tested in order to find three positives.

16. **(5090*)** If Y is a Negative Binomial distribution with parameters p and r, let $X = Y - r$, the number of trials until the rth failure. Show that

$$P(X = x) = \binom{x+r-1}{r-1} p^r (1-p)^x = \binom{x+r-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots$$