Homework 5

Due Thursday, February 27

- 1. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be
 - (a) at least 2 such accidents in the next month?
 - (b) at most 1 accident in the next month?
- 2. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. The number of claims filed has a Poisson distribution. Calculate the variance of the number of claims filed.
- 3. Suppose that the number of accidents occurring on a highway each day is a Poisson random variable with parameter $\lambda = 3$.
 - (a) Find the probability that 3 or more accidents occur today.
 - (b) Find the probability that 3 or more accidents occur today given that at least 1 accident occurred today already.
- 4. The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 5$. Suppose that a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 3$ for 75 percent of the population. For the other 25 percent of the population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 2 colds in that time, how likely is it that the drug is beneficial for him or her?
- 5. A parking lot has two entrances. Cars arrive at entrance I according to a Poisson distribution at an average of three per hour and at entrance II according to a Poisson distribution at an average of four per hour. What is the probability that a total of three cars will arrive at the parking lot in a given hour? (Assume that the numbers of cars arriving at the two entrances are independent.)
- 6. A store owner has overstocked a certain item and decides to use the following promotion to decrease the supply. The item has a marked price of \$100. For each customer purchasing the item during a particular day, the owner will reduce the price by a factor of one-half. Thus, the first customer will pay \$50 for the item, the second will pay \$25, and so on. Suppose that the number of customers who purchase the item during the day has a Poisson distribution with mean 2. Find the expected cost of the item at the end of the day. (Hint: The cost at the end of the day is $100(1/2)^X$, where X is the number of customers who have purchased the item.)

- 7. Consider a roulette wheel consisting of 38 numbers 1 through 36, 0, and double 0 (00). If Smith always bets that the outcome will be one of the numbers 1 through 12, what is the probability that
 - (a) Smith will lose his first 5 bets?
 - (b) his first win will occur on his fourth bet?
- 8. About six months into George W. Bush's second term as president, a Gallup poll indicated that a near record (low) level of 41% of adults expressed "a great deal" or "quite a lot" of confidence in the U.S. Supreme Court. Suppose that you conducted your own telephone survey at that time and randomly called people and asked them to describe their level of confidence in the Supreme Court. Find the probability distribution for X, the number of calls until the first person is found who does not express "a great deal" or "quite a lot" of confidence in the U.S. Supreme Court.
- 9. A performer has a constant probability, less than 0.5, of having an accident in any particular performance. The occurrence of an accident in any one performance is independent of the occurrence of an accident in all other performances. The probability that the performer's first accident occurs in the second performance is 0.16. Calculate the probability that the performer's first accident occurs in the fourth performance.
- 10. If X is a geometric random variable with success probability p,
 - (a) show that $P(X > a) = (1 p)^a$.
 - (b) using the previous part, show that P(X = x + k | X > x) = P(X = k).
- 11. If X has a geometric distribution with success probability 0.3, what is the largest value, a, such that $P(X > a) \ge 0.1$?
- 12. The employees of a firm that manufactures insulation are being tested for indications of asbestos in their lungs. The firm is requested to send three employees who have positive indications of asbestos on to a medical center for further testing. If 40% of the employees have positive indications of asbestos in their lungs, find the probability that ten employees must be tested in order to find three positives.
- 13. The telephone lines serving an airline reservation office are all busy about 60% of the time.
 - (a) If you are calling this office, what is the probability that you will complete your call on the first try? The second try? The third try?
 - (b) If you and a friend must both complete calls to this office, what is the probability that a total of four tries will be necessary for both of you to get through?
- 14. Each time a hurricane arrives, a new home has a 0.4 probability of experiencing damage. The occurrences of damage in different hurricanes are mutually independent. Formulate a probability of the number of hurricanes it takes for the home to experience damage from two hurricanes. Calculate such probabilities from two to five.
- 15. Suppose that a batch of 100 items contains 6 that are defective and 94 that are not defective. If X is the number of defective items in a randomly drawn sample of 10 items from the batch, find (a) P(X = 0) and (b) P(X > 2).

- 16. A state is starting a lottery game. To enter this lottery, a player uses a machine that randomly selects six distinct numbers from among the first 30 positive integers. The lottery randomly selects six distinct numbers from the same 30 positive integers. A winning entry must match the same set of six numbers that the lottery selected. The entry fee is 1, each winning entry receives a prize amount of 500,000, and all other entries receive no prize. Calculate the probability that the state will lose money, given that 800,000 entries are purchased.
- 17. Consider a binomial experiment for n = 20, p = 0.05. Calculate the binomial probabilities for X = 0, 1, 2. Calculate the same probabilities by using the Poisson approximation with $\lambda = np$. Compare.
- 18. Repeat Problem 16, using the Poisson approximation.
- 19. (5090*) A company has purchased a policy that will compensate for the loss of revenue due to severe weather events. The policy pays 1000 for each severe weather event in a year after the first two such events in that year. The number of severe weather events per year has a Poisson distribution with mean 1. Calculate the expected amount paid to this company in one year.
- 20. (5090*) If X has a geometric distribution with success probability p, what is the probability of X being an odd integer?
- 21. (5090*) If X is a Negative Binomial distribution with parameters p and r, let Y = X r, the number of trials until the rth failure. Show that

$$P(Y=y) = {\binom{y+r-1}{r-1}}p^r(1-p)^y = {\binom{y+r-1}{y}}p^r(1-p)^y, \quad y = 0, 1, 2, \dots$$