Homework 7

Due Thursday, March 21

1. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\beta = 2$. What is
   (a) the probability that a repair time exceeds 2 hours?
   (b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?

2. Jones figures that the total number of miles (in thousands of miles) that an auto can be driven before it would need to be junked is an exponential random variable with parameter $\beta = 20$. Smith has a used car that has 10,000 miles on it. If Jones purchases the car, what is the probability that she would get at least 20,000 additional miles out of it? Repeat under the assumption that the lifetime mileage of the car is not exponentially distributed, but rather is (in thousands of miles) uniformly distributed over (0, 40).

3. Find the median of $Y \sim \text{Exponential}(\beta)$.

4. If Y has Gamma(α, β) distribution, show that, for any integer k,

$$E(Y^k) = \frac{\Gamma(\alpha + k)}{\Gamma(\alpha)} \beta^k$$

5. The percentage of impurities per batch in a chemical product is a random variable Y with density function $f_Y(y) = 12y^2(1 - y)$, $0 < y < 1$. A batch with more than 40% impurities cannot be sold. Determine the probability that a randomly selected batch cannot be sold because of excessive impurities. Also, what is the expected percentage of impurities per batch?

6. If Y has the pdf $f_Y(y) = ye^{-y}$, $y > 0$, show, by performing the integration, that

$$M_Y(t) = E(e^{tY}) = \frac{1}{(1 - t)^2}$$

7. Suppose that Y has the pdf

$$f_Y(y) = \begin{cases} 1 + y, & -1 < y < 0 \\ 1 - y, & 0 \leq y < 1 \end{cases}$$

   Find the mgf of Y.

8. If $M_Y(t) = e^{t^2/2}$, find $E(Y^3)$ by differentiating the mgf.

9. **(5090*)** Show that $P(Y \geq a) \leq e^{-at} M_Y(t)$, for $t > 0$.

10. **(5090*)** Show that each of the following belongs to an exponential family.

   (a) $Y \sim \text{Poisson}(\theta)$

   (b) $Y \sim N(\theta, 1)$

   (c) $Y \sim \text{Beta}(2, \theta)$

   (Please see the notes on Exponential Families on course webpage)