Homework 8

Due Thursday, March 28

1. Two fair dice are rolled. Find the joint probability function of Y_1 and Y_2, when Y_1 is the value on the first die and Y_2 is the larger of the two values. Assume that the ties are allowed (e.g., the event $\{Y_1 = 2, Y_2 = 2\}$ has outcomes (2,1) and (2,2)).

2. Suppose that balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let Y_i equal 1 if the ith ball selected is white, and let it equal 0 otherwise. Give the joint probability function of (Y_1, Y_2).

3. The joint density (pdf) of Y_1 and Y_2 is given by

$$f(y_1, y_2) = e^{-(y_1+y_2)}, \quad 0 < y_1 < \infty,\ 0 < y_2 < \infty$$

   (a) Find $P(Y_1 < 1, Y_2 > 5)$.
   (b) Find $P(Y_1 + Y_2 < 3)$.
   (c) Find $P(Y_1 < Y_2)$.

4. Show that $f(y_1, y_2) = 1/y_1$, $0 < y_2 < y_1 < 1$, is a joint density function of Y_1 and Y_2. (HINT: The order in which you want to integrate makes a difference).

5. The joint density function of Y_1 and Y_2 is given by

$$f(y_1, y_2) = y_1 e^{-y_1(y_2+1)}, \quad y_1 > 0,\ y_2 > 0$$

   Find the conditional density of Y_1, given $Y_2 = y_2$.

6. The joint density of Y_1 and Y_2 is

$$f(y_1, y_2) = c(y_1^2 - y_2^2)e^{-y_1}, \quad 0 \le y_1 < \infty,\ -y_1 \le y_2 \le y_1$$

   Find the conditional density of Y_2, given $Y_1 = y_1$.

7. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = k(1 - y_2), \quad 0 \le y_1 \le y_2 \le 1$$

   (a) Find the value of k that makes this a probability density function.
   (b) Find $P(Y_1 \le 3/4, Y_2 \ge 1/2)$.
   (c) Find the marginal density functions for Y_1 and Y_2.
   (d) Find the conditional density function of Y_1 given $Y_2 = y_2$.
   (e) Find the conditional density function of Y_2 given $Y_1 = y_1$.
   (f) Find $P(Y_2 \ge 3/4 | Y_1 = 1/2)$.

8. The joint probability function of Y_1 and Y_2 is given by

$$p(1,1) = P(Y_1 = 0, Y_2 = 1) = \frac{1}{8}, \qquad p(1,2) = P(Y_1 = 0, Y_2 = 2) = \frac{1}{4},$$
$$p(2,1) = P(Y_1 = 2, Y_2 = 1) = \frac{1}{8}, \qquad p(2,2) = P(Y_1 = 2, Y_2 = 2) = \frac{1}{2}.$$

   (a) Compute the conditional probability function of Y_1, given $Y_2 = i$, $i = 1, 2$
   (b) Are Y_1 and Y_2 independent?

9. Are Y_1 and Y_2 independent if the joint density of Y_1 and Y_2 is given by

   (a)
$$f(y_1, y_2) = y_1 e^{-(y_1+y_2)}, \quad 0 < y_1 < \infty,\ 0 < y_2 < \infty$$

   (b)
$$f(y_1, y_2) = 2, \quad 0 < y_1 < y_2,\ 0 < y_2 < 2$$

10. The joint density function of Y_1 and Y_2 is

$$f(y_1, y_2) = y_1 + y_2, \quad 0 < y_1 < 1,\ 0 < y_2 < 1$$

   (a) Are Y_1 and Y_2 independent?
   (b) Find the marginal densities (pdfs).
   (c) Find $P(Y_1 + Y_2 < 1)$.

11. **(5090*)** A and B agree to meet at certain place between 1 and 2 PM. Suppose that they arrive at the meeting place independently and randomly during the hour. Find the distribution of length of time that A waits for B (the waiting time is 0 if B arrives before A).

12. **(5090*)** Suppose that Y_1 and Y_2 are independent Poisson distributed random variables with means λ_1 and λ_2, respectively. Let $W = Y_1 + Y_2 \sim \text{Poisson}(\lambda_1 + \lambda_2)$ (we will prove this fact in Chapter 6). Show that the conditional distribution of Y_1 given $W = w$, is a binomial distribution with $n = w$ and $p = \lambda_1/(\lambda_1 + \lambda_2)$.