Homework 9
Due Thursday, April 4

1. The joint density function of Y_1 and Y_2 is

$$f(y_1, y_2) = y_1 + y_2, \quad 0 < y_1 < 1, \ 0 < y_2 < 1$$

   Find $E(Y_1)$ and $E(Y_2)$.

2. Let (Y_1, Y_2) have the joint pdf

$$f(y_1, y_2) = 4y_1y_2, \quad 0 < y_1 < 1, \ 0 < y_2 < 1$$

   Find

   (a) $E(Y_1)$
   (b) $\mathrm{Var}(Y_1)$
   (c) $E(Y_1 - Y_2)$

3. If Y_1 and Y_2 have joint density function

$$f(y_1, y_2) = \frac{1}{y_2}, \quad 0 < y_1 < y_2 < 1$$

   find

   (a) $E(Y_1Y_2)$
   (b) $E(Y_1)$
   (c) $E(Y_2)$
   (d) $\mathrm{Cov}(Y_1, Y_2)$.

4. Let (Y_1, Y_2) have the joint pdf

$$f(y_1, y_2) = 1, \quad 0 < y_1 < 2, \ 0 < y_2 < 1, \ 2y_2 < y_1.$$

   Find $E(Y_1 - Y_2)$.

5. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = 6(1 - y_2), \quad 0 \le y_1 \le y_2 \le 1$$

   (a) Show that $\mathrm{Cov}(Y_1, Y_2) = 1/40$
   (b) Find $\mathrm{Var}(Y_1 - 3Y_2)$

6. If Y_1 and Y_2 are random variables, and a and b are constants, show that

   (a) $\text{Cov}(Y_1, Y_2) = \text{Cov}(Y_2, Y_1)$
   (b) $\text{Cov}(aY_1, Y_2) = a\text{Cov}(Y_1, Y_2)$
   (c) $\text{Cov}(aY_1, bY_2) = ab\text{Cov}(Y_1, Y_2)$
   (d) $\text{Cov}(aY_1, Y_1 + Y_2) = a\text{Var}(Y_1) + a\text{Cov}(Y_1, Y_2)$
   (e) $\text{Cov}(aY_1 + bY_2, Y_1 + Y_2) = a\text{Var}(Y_1) + b\text{Var}(Y_2) + (a + b)\text{Cov}(Y_1, Y_2)$

7. An insurance policy pays a total medical benefit consisting of two parts for each claim. Let X represent the part of the benefit that is paid to the surgeon, and let Y represent the part that is paid to the hospital. The variance of X is 5,000, the variance of Y is 10,000, and the variance of the total benefit, $X + Y$, is 17,000. Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim. Calculate the variance of the total benefit after these revisions have been made.

8. **(5090*)** Let X denote the size of a surgical claim and let Y denote the size of the associated hospital claim. An actuary is using a model in which

$$E(X) = 5,\ E(X^2) = 27.4,\ E(Y) = 7,\ E(Y^2) = 51.4,\ \text{Var}(X + Y) = 8$$

   Let $C_1 = X + Y$ denote the size of the combined claims before the application of a 20% surcharge on the hospital portion of the claim, and let C_2 denote the size of the combined claims after the application of that surcharge. Calculate $\text{Cov}(C_1, C_2)$.

9. **(5090*)** Let (Y_1, Y_2) have the joint pdf

$$f(y_1, y_2) = 2, \quad 0 < y_1 < 1,\ 0 < y_2 < 1,\ 0 < y_1 + y_2 < 1$$

   Find $\text{Var}(Y_1 + Y_2)$