Homework 10

Due Thursday, April 18

1. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \frac{1}{y_2}, \quad 0 < y_1 < y_2 < 1$$

   Find $E(Y_1|Y_2 = y_2)$.

2. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = 6(1 - y_2), \quad 0 \le y_1 \le y_2 \le 1$$

   Find $E(Y_1|Y_2 = y_2)$.

3. The joint density of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \frac{e^{-y_2}}{y_2}, \quad 0 < y_1 < y_2 < \infty$$

   Compute $E(Y_1^3|Y_2 = y_2)$.

4. Show that, if Y_1 and Y_2 are independent, then

$$E(Y_1|Y_2 = y_2) = E(Y_1)$$

   for all y_2.

5. If Y_1, Y_2, Y_3, Y_4, Y_5 are iid exponential random variables with the parameter β, compute

$$P(\min(Y_1, Y_2, Y_3, Y_4, Y_5) > a)$$

6. Let $Y_1, \ldots, Y_n$ be iid random variables, each with pdf

$$f_Y(y) = e^{-(y-\theta)}, \quad y > \theta$$

   Find the pdf of $Y_{(1)} = \min\{Y_1, \ldots, Y_n\}$.

7. Consider a sample of size 5 from a uniform distribution over (0, 1). Compute the probability that the median is in the interval (1/4, 3/4).

8. Let $Y_1, \ldots, Y_n$ be iid Uniform$(0, \theta)$. Find the pdf of $Y_{(k)}$, and show that it follows a beta distribution if $\theta = 1$.

9. **(5090*)** Suppose that $Y_2|Y_1 \sim \text{Poisson}(Y_1)$ and $Y_1 \sim \text{Gamma}(\alpha, \beta)$, so that

$$P(Y_2 = y_2|Y_1 = y_1) = \frac{e^{-y_1} y_1^{y_2}}{y_2!}, \quad y_2 = 0, 1, 2, \ldots$$

and that

$$f_{Y_1}(y_1) = \frac{y_1^{\alpha-1} e^{-y_1/\beta}}{\Gamma(\alpha)\beta^\alpha}, \quad y_1 > 0.$$

(a) Find $E(Y_2)$ and $\text{Var}(Y_2)$ (no need to find $f_{Y_2}(y_2)$ for this).

(b) Assuming that we can write $f(y_2|y_1) = P(Y_2 = y_2|Y_1 = y_1)$, find $f(y_1, y_2)$ and $f(y_1|y_2)$.

For the below problems, you will need to use the formula for the joint pdf of $(Y_{(j)}, Y_{(k)})$

$$\begin{aligned} & f_{Y_{(j)},Y_{(k)}}(y_j, y_k) \\ = & \frac{n!}{(j-1)!(k-1-j)!(n-k)!}[F_Y(y_j)]^{j-1}[F_Y(y_k) - F_Y(y_j)]^{k-1-j}[1 - F_Y(y_k)]^{n-k} f_Y(y_j) f_Y(y_k) \end{aligned}$$

where $y_j < y_k$ (compare with WMS formula on page 337).

10. **(5090*)** Suppose that $Y_1, \ldots, Y_n$ are iid Uniform$(0, \theta)$. Find the joint pdf of $(Y_{(1)}, Y_{(n)})$

11. **(5090*)** Let Y_1 and Y_2 be iid Uniform$(0, 1)$. Find $P(2Y_{(1)} < Y_{(2)})$.