Homework 1

Due Tuesday, January 23

1. Let Y be a random variable with probability density function given by

$$f_Y(y) = 2(1-y), \quad 0 < y < 1.$$

Using the cdf method, find the density functions of

(a) $U_1 = 2Y - 1$

(b) $U_2 = 1 - 2Y$

(c) $U_3 = Y^2$

2. Repeat Problem 1 using the pdf method.

3. If Y is uniformly distributed over (-1, 1), find

(a) $P(|Y| > 1/2)$

(b) The pdf of $|Y|$

4. Suppose that Y is an exponential random variable (the pdf of Y is given by

$$f_Y(y) = \frac{1}{\beta} e^{-y/\beta}, \quad y > 0).$$

If the parameter $\beta = 1$, compute the density of the random variable U defined by $U = \ln Y$. Try both the cdf and pdf methods.

5. Let Y have a gamma distribution with parameters α and β, so that the pdf of Y is

$$f_Y(y) = \frac{1}{\Gamma(\alpha)\beta^\alpha} y^{\alpha-1} e^{-y/\beta}, \quad y > 0.$$

Suppose that $c > 0$ is a constant. Find the pdf of $U = cY$.

6. Suppose that $Y \sim N(0,1)$ (standard normal) and $U = Y^2$. Find the pdf of U, and show that U follows a gamma distribution (HINT: $\Gamma(1/2) = \sqrt{\pi}$).

7. Let Y have the pdf

$$f_Y(y) = \frac{e^{-y}}{(1+e^{-y})^2}, \quad -\infty < y < \infty.$$

Let $U = e^{-Y}$. Find the pdf of U.

8. Assume that Y has a beta distribution with parameters α and β, so that the pdf of Y is

$$f_Y(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 < y < 1.$$

Find the pdf of $U = 1 - Y$, along with $E(U)$ and $\mathrm{Var}(U)$