Homework 2
Due Tuesday, January 30

1. Suppose that (Y_1, Y_2) have the joint pdf $f_{Y_1,Y_2}(y_1, y_2) = e^{-y_1}$, $0 < y_2 < y_1 < \infty$. Find the pdf of $U = Y_1 - Y_2$ using the cdf method.

2. Suppose that Y_1 and Y_2 are independent, with densities $f_{Y_1}(y_1) = 6y_1(1 - y_1)$, $0 < y_1 < 1$, and $f_{Y_2}(y_2) = 3y_2^2$, $0 < y_2 < 1$, respectively. Find the pdf of $U = Y_1Y_2$ using the cdf method.

3. Suppose that a machine has two independent components and that each component has a lifetime governed by the exponential distribution with mean 1. Find the pdf for the average lifetime of the two components, using the cdf method.

4. Y_1 and Y_2 have joint density function
$$f_{Y_1,Y_2}(y_1, y_2) = \frac{1}{y_1^2 y_2^2}, \quad y_1 \geq 1,\ y_2 \geq 1$$
Compute the joint density function of $U_1 = Y_1Y_2$ and $U_2 = Y_1/Y_2$.

5. If Y_1 and Y_2 are independent and identically distributed uniform random variables on (0, 1), compute the joint density of $U_1 = Y_1 + Y_2$, $U_2 = Y_1/Y_2$.

6. If Y_1 and Y_2 are independent and identically distributed exponential random variables with $\beta = 1$, compute the joint density of $U_1 = Y_1$, $U_2 = Y_1/Y_2$.

7. If Y_1 and Y_2 are independent exponential random variables, each having parameter $\beta = 1/\lambda$, find the joint density function of $U_1 = Y_1 + Y_2$ and $U_2 = e^{Y_1}$.

8. Let Y_1 and Y_2 be independent random variables with pdfs $f_{Y_1}(y_1)$, $-\infty < y_1 < \infty$, and $f_{Y_2}(y_2)$, $-\infty < y_2 < \infty$, respectively. Let $U_1 = Y_1 + Y_2$ and $U_2 = Y_2$. Show that the pdf of U_1 is given by
$$f_{U_1}(u_1) = \int f_{Y_1}(u_1 - u_2) f_{Y_2}(u_2)\, du_2$$

9. Repeat Problem 3 using the pdf (Jacobian) method.

10. If Y has the pdf $f_Y(y) = ye^{-y}$, $y > 0$, show that
$$M_Y(t) = E(e^{tY}) = \frac{1}{(1-t)^2} = (1-t)^{-2}$$
by performing the integration.

11. Suppose that Y has the pdf
$$f_Y(y) = \begin{cases} 1 + y, & -1 < y < 0 \\ 1 - y, & 0 \leq y < 1 \end{cases}$$
Find the mgf of Y.

12. If $M_Y(t) = e^{t^2/2}$, find $E(Y^3)$ by differentiating the mgf of Y.