

Homework 5

Due Tuesday, February 27

1. If Y_1, Y_2, Y_3, Y_4, Y_5 are iid exponential random variables with the parameter β , compute

$$P(\min(Y_1, Y_2, Y_3, Y_4, Y_5) > a)$$

For Problems 2 and 3, you may need to use the formula for the pdf of $Y_{(k)}$

$$f_{Y_{(k)}}(y) = \frac{n!}{(k-1)!(n-k)!} [F_Y(y)]^{k-1} [1 - F_Y(y)]^{n-k} f_Y(y)$$

(compare with WMS formula on page 336).

2. Let Y_1, \dots, Y_n be iid random variables, each with pdf $f_Y(y) = e^{-(y-\theta)}$, $y > \theta$. Find the pdf of $Y_{(1)} = \min\{Y_1, \dots, Y_n\}$.
3. Let Y_1, \dots, Y_n be iid Uniform(0, θ). Find the pdf of $Y_{(k)}$, and show that it follows a beta distribution if $\theta = 1$.

For Problems 4 to 6, you will need to use the formula for the joint pdf of $(Y_{(j)}, Y_{(k)})$

$$\begin{aligned} & f_{Y_{(j)}, Y_{(k)}}(y_j, y_k) \\ &= \frac{n!}{(j-1)!(k-1-j)!(n-k)!} [F_Y(y_j)]^{j-1} [F_Y(y_k) - F_Y(y_j)]^{k-1-j} [1 - F_Y(y_k)]^{n-k} f_Y(y_j) f_Y(y_k) \end{aligned}$$

where $y_j < y_k$ (compare with WMS formula on page 337).

4. **(5880*)** Suppose that Y_1, \dots, Y_n are iid Uniform(0, θ).
- (a) Find the joint pdf of $(Y_{(1)}, Y_{(n)})$
- (b) Find the joint pdf of $U_1 = Y_{(1)}/Y_{(n)}$ and $U_2 = Y_{(n)}$.
5. **(5880*)** Let Y_1, \dots, Y_n be iid Uniform(0, 1). Find the pdf of the sample range, $R = Y_{(n)} - Y_{(1)}$ (Hint: let $S = Y_{(1)}$ to find the joint pdf of (R, S) before finding the pdf of the sample range).
6. **(5880*)** Let Y_1 and Y_2 be iid Uniform(0, 1). Find $P(2Y_{(1)} < Y_{(2)})$.