

Homework 8

Due Tuesday, April 2

1. If $Y_1, Y_2, \dots, Y_n \sim \text{iid Geometric}(p)$, show that \bar{Y} is a sufficient statistic for p .
2. Let Y_1, Y_2, \dots, Y_n denote a random sample from a Poisson distribution with parameter θ . Find the MVUE of θ .
3. Let Y_1, Y_2, \dots, Y_n be iid $\text{Gamma}(\alpha, \beta)$. Suppose that α is known. Find the MVUE of β .
4. Let Y_1, Y_2, \dots, Y_n be iid $N(\theta, 1)$. Find the MVUE of θ^2 .
5. Let Y_1, \dots, Y_n be iid random variables, each with pdf $f_Y(y) = e^{-(y-\theta)}$, $y > \theta$. Find a sufficient statistic for θ .
6. Let Y_1, Y_2, \dots, Y_n be random sample whose density is given by

$$f_Y(y) = \frac{\alpha y^{\alpha-1}}{\theta^\alpha}, \quad 0 \leq y \leq \theta$$

where $\alpha > 0$ is a known fixed value, but θ is unknown. Find a sufficient statistic for θ .

7. Let Y_1, Y_2, \dots, Y_n be iid $\text{Gamma}(\alpha, \beta)$. Find sufficient statistics for (α, β) .
8. **(5880*)** Let Y_1, Y_2, \dots, Y_n be random sample whose density is given by

$$f_Y(y) = \frac{\alpha y^{\alpha-1}}{\theta^\alpha}, \quad 0 \leq y \leq \theta$$

where now both $\alpha > 0$ and $\theta > 0$ are unknown. Find sufficient statistics for (α, θ) .

(The following two problems will require the independent reading of Complete and Sufficient Statistics notes, on course webpage)

9. **(5880*)** Let Y_1, Y_2, \dots, Y_n be iid with pdf

$$f_Y(y) = \frac{\theta}{(1+y)^{1+\theta}}, \quad y > 0, \quad \theta > 0$$

Find a complete sufficient statistic for θ .

10. **(5880*)** Let Y_1, Y_2, \dots, Y_n be iid with pdf

$$f_Y(y) = \theta e^{-\theta y}, \quad y > 0, \quad \theta > 0$$

Show that the UMVUE of θ is given by

$$\frac{n-1}{\sum_{i=1}^n Y_i}$$