Homework 9

Due Tuesday, April 9

1. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with parameter θ. Find the method of moments estimator for θ.

2. Suppose that $Y_1, Y_2, \ldots, Y_n \sim$ iid $N(\mu, \sigma^2)$. Find the method of moments estimators for μ and σ^2.

3. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a distribution with pdf

$$f_Y(y) = \theta k^\theta y^{-(\theta+1)}, \quad y > k, \theta > 1$$

Suppose that k is known. Find the method of moments estimator for θ.

4. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with parameter θ.
   (a) Find the MLE of θ.
   (b) Find the MLE of $P(Y = 0)$.

5. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a gamma distribution with parameters α and β. Suppose that α is known.
   (a) Find the MLE of β.
   (b) Find the MLE of $E(Y)$.

6. Let $Y_1, \ldots, Y_n$ be iid random variables, each with pdf

$$f_Y(y) = \theta^2 y e^{-\theta y}, \quad y > 0$$

Find the MLE of θ.

7. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a pdf

$$f_Y(y) = \frac{r y^{r-1}}{\theta} e^{-y^r/\theta}, \quad y > 0$$

where $\theta > 0$ is the parameter and r is known.
   (a) Find a sufficient statistic of θ.
   (b) Find the MLE of θ.

8. Let $Y_1, \ldots, Y_n$ be iid random variables, each with pdf

$$f_Y(y) = \frac{\theta}{y^2}, \quad \theta \le y < \infty$$

Find (1) sufficient statistic, (2) MLE, and (3) method of moment estimator of θ.

9. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a pdf

$$f_Y(y) = \frac{2\theta^2}{y^3}, \quad \theta \le y < \infty$$

Find the MLE of θ^2.

10. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a pdf

$$f_Y(y) = \frac{3y^2}{\theta^3}, \quad 0 \le y \le \theta$$

Find the MLE of θ.

(*The following two problems require the calculation of Z_{MLE}, defined as*

$$Z_{MLE} = \frac{t(\hat{\theta}) - t(\theta)}{\sqrt{\left[\frac{\partial t(\theta)}{\partial \theta}\right]^2 \Big/ I(\theta)}}$$

where $I(\theta)$ is the Fisher information; see Section 9.8 of WMS).

11. **(5880*)** Let $Y_1, \ldots, Y_n$ denote a random sample from an exponential distribution with parameter θ. Find Z_{MLE} for (a) $t(\theta) = \theta$ and (b) $t(\theta) = \theta^2$.

12. **(5880*)** Let $Y_1, \ldots, Y_n$ denote a random sample from a Poisson distribution with parameter λ. Find Z_{MLE} for $t(\lambda) = P(Y = 0)$.