Quiz Solution

1. First, note that

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}, \quad -\infty < x < \infty$$

Since

$$F_U(u) = P(U \le u) = P(|Y| \le u) = P(-u \le Y \le u) = P(Y \le u) - P(Y \le -u) = F_Y(u) - F_Y(-u)$$

we have that

$$\begin{aligned} f_U(u) &= \frac{d}{du} F_U(u) = \frac{d}{du}[F_Y(u) - F_Y(-u)] = f_Y(u) + f_Y(-u) \\ &= \frac{1}{\sqrt{2\pi}} e^{-u^2/2} + \frac{1}{\sqrt{2\pi}} e^{-u^2/2} = \sqrt{\frac{2}{\pi}} e^{-u^2/2}, \quad u > 0 \end{aligned}$$

2. Since $M_{Y_1}(t) = \left(1/3 + 2/3e^t\right)^3 = \left((1 - 2/3) + 2/3e^t\right)^3$ is the mgf of $Y_1 \sim \text{Binomial}(3, 2/3)$, and $M_{Y_2}(t) = \exp(2e^t - 2) = \exp(2(e^t - 1))$ is the mgf of $Y_2 \sim \text{Poisson}(2)$, and Y_1 and Y_2 are independent, it follows that

$$E(Y_1 Y_2) = E(Y_1)E(Y_2) = (np)\lambda = (3 \cdot (2/3))2 = 4$$