Math 142, Problem Set #1
(due **in class** Fri., 1/31/14)

Note: To get full credit for a problem, it is not enough to give the right answer; you must explain your reasoning.

Also note that this is a two-page assignment.

Stewart, section 5.1, problems 14, 18, and 24. (For problem 14, give four estimates of the distance travelled: an upper bound that divides the time into three equal intervals, a lower bound that divides the time into three equal intervals, an upper bound that divides the time into six equal intervals, and a lower bound that divides the time into six equal intervals. Use these numbers to argue that one gets more accurate estimates of the distance travelled when one divides the time into more, and shorter, intervals.)

Stewart, Appendix B, problem 47.

Also:

A. Use the Constant Sequence Principle discussed in class to prove formula (e) in Theorem 3 on page A13 of Stewart.

B. Use mathematical induction (as described in Appendix B) to prove formula (e) in Theorem 3 on page A13 of Stewart.

C. Near the end of Example 2 (section 5.1), Stewart writes "It can be shown that the lower approximating sums also approach $\frac{1}{3}$, that is, $\lim_{n\to\infty} L_n = \frac{1}{3}$." Prove this.

D. Using the method Stewart uses in example 2 of section 5.1, compute the area under the parabola $y = x^2$ from $x = 0$ to $x = 2$.

E. In class, we saw that the area of the region that lies under the graph of $f(x) = e^{-x}$ between $x = 0$ and $x = 2$ can be written as $A = \lim_{n\to\infty} L_n$ where

$$L_n = \frac{2}{n}\,\frac{1 - e^{-2}}{1 - e^{-2/n}}.$$

Compute A by replacing n by $1/x$ and using L'Hospital's Rule.

F. In class, we proved that $\sum_{k=1}^{n} k^4 > n^5/5$ for all $n \geq 1$. Show that $\sum_{k=1}^{n} k^4 < (n+1)^5/5$ for all $n \geq 1$.

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), **and to write down on the timesheet how many minutes you spent on each problem** (this doesn't need to be exact).