Math 142, Problem Set #6
(due **in class** Fri., 3/14/14)

Note: To get full credit for a problem, it is not enough to give the right answer; you must explain your reasoning.

Also note that this is a two-page assignment.

Stewart, section 6.6, problems 2, 6, 8, 24, 32, 50, 52, 59, 60, 62.
Hints:

- For problem 50, you may find it useful to use the result of problem 3.7.41 and the result of problem 6.1.36.
- For problem 52, you are supposed to show that if $\int_{-\infty}^{a} f(x)\,dx$ and $\int_{a}^{\infty} f(x)\,dx$ exist as proper integrals, then $\int_{-\infty}^{b} f(x)\,dx$ and $\int_{b}^{\infty} f(x)\,dx$ exist as proper integrals, and satisfy
$$\int_{-\infty}^{a} f(x)\,dx + \int_{a}^{\infty} f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx + \int_{b}^{\infty} f(x)\,dx.$$
Note that Definition 1 makes use of this Exercise, so you cannot simply cite Definition 1; that would be circular reasoning.
- For problem 59, use integration by parts, with $u = x$ and $dv = xe^{-x^2}\,dx$.
- For problem 60, you do not need to evaluate either integral. You just need to explain why they are equal to each other, by showing that the two integrals can be interpreted as two different ways of calculating the area of a single region.

Also:

A. (a) Suppose we try to integrate $1/x$ by parts, taking $u = 1/x$ and $dv = dx$. We have $du = (-1/x^2)\,dx$ and $v = x$, so
$$\int \frac{1}{x}\,dx = \frac{1}{x}\cdot x - \int \frac{-1}{x^2}\cdot x\,dx = 1 + \int \frac{1}{x}\,dx$$
Canceling the integral from both sides, we get the disconcerting result that $0 = 1$. What went wrong?

(b) What happens if we replace the indefinite integrals by definite integrals, that is, if we try to calculate

$$\int_a^b \frac{1}{x}\, dx$$

by this method? (You may assume that $0 < a < b$.)

B. Consider the region bounded by the line $x = 0$, the line $x = 1$, the line $y = 0$, and the curve $y = \ln x$. Evaluate the area of this region in two different ways: first by writing it as an improper integral of type II with respect to x, and second by writing it as an improper integral of type I with respect to y. (Hint: For the former approach to computing the area, you may find Example 6 from section 3.7 helpful.)

C. Show that $\int_0^1 (\frac{d}{dx}\sqrt{1-x^2})\, dx = \sqrt{1-x^2}|_0^1$. Explain why this does not follow from the Evaluation Theorem of section 5.3.

D. Does the improper integral $\int_0^\infty (\frac{\sin x}{x})^2\, dx$ converge? Explain by giving separate consideration to the integrals $\int_0^1 (\frac{\sin x}{x})^2\, dx$ and $\int_1^\infty (\frac{\sin x}{x})^2\, dx$.

E. (a) What is $\int_{-\infty}^{\infty} \sin x\, dx$?

(b) What is $\lim_{t\to\infty} \int_{-t}^{t} \sin x\, dx$?

(c) Note that your answers for (a) and (b) are different. Explain why this is compatible with what we know about definite integrals of type I.

F. Does $\int_2^\infty \frac{1}{\sqrt{x}-1}\, dx$ converge? Explain. (Note that the denominator is $\sqrt{x}-1$, not $\sqrt{x-1}$.)

Please don't forget to write down **who you worked on the assignment with** (if nobody, then write "I worked alone"), **and how much time you spent on each problem** (this doesn't need to be exact).