Math 192r, Problem Set #16
(due 11/20/01)

1. Use Dodgson condensation to prove the Vandermonde determinant formula
$$\det(M) = \prod_{1 \le i < j \le n} (x_j - x_i)$$
where M is the n-by-n matrix whose i, jth entry (for $1 \le i, j \le n$) is x_j^{i-1}.

2. Using Dodgson condensation, Lindstrom's lemma, and the bijection between tilings and routings discussed in class, prove that for all $a, b, c \ge 0$, the number of ways to tile an a, b, c, a, b, c semiregular hexagon with unit rhombuses is equal to
$$\frac{H(a+b+c)H(a)H(b)H(c)}{H(a+b)H(a+c)H(b+c)}$$
where $H(0) = H(1) = 1$ and $H(n) = 1!2!3! \cdots (n-1)!$ for $n > 1$.

For both of these problems, you should use only the properties of the determinant that were discussed in lecture (or that you prove yourself).