1. Use Dodgson condensation to prove the Vandermonde determinant formula

$$\det(M) = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

where $M$ is the $n$-by-$n$ matrix whose $i,j$th entry (for $1 \leq i, j \leq n$) is $x_j^{i-1}$.

2. Using Dodgson condensation, Lindstrom’s lemma, and the bijection between tilings and routings discussed in class, prove that for all $a, b, c \geq 0$, the number of ways to tile an $a, b, c, a, b, c$ semiregular hexagon with unit rhombuses is equal to

$$\frac{H(a + b + c)H(a)H(b)H(c)}{H(a + b)H(a + c)H(b + c)}$$

where $H(0) = H(1) = 1$ and $H(n) = 1!2!3! \cdots (n - 1)!$ for $n > 1$.

For both of these problems, you should use only the properties of the determinant that were discussed in lecture (or that you prove yourself).