

Math 192r, Problem Set #21  
(due 12/13/01)

1. For  $n \geq 0$  let  $A(n) = \sum_{n/2 \leq k \leq n} 2^k$  (where the sum is only over integer values of  $k$ ), so that  $A(0) = 1$ ,  $A(1) = 2$ ,  $A(2) = 6$ , etc. Extend  $A(n)$  to the negative domain in two different ways, and check that they agree: first, by finding a formula for  $A(n)$  when  $n$  is positive; and second, by applying the polytope reciprocity theorem.
2. For  $n \geq 0$ , let  $f(n)$  be the number of integer sequences of length  $n + 1$  consisting of 1's, 2's, 3's, and 4's, such that the first term is 1, the last term is 1, and any two consecutive terms differ by 0 or  $\pm 1$ . Thus  $f(0) = 1$ ,  $f(1) = 1$ ,  $f(2) = 2$ ,  $f(3) = 4$ ,  $f(4) = 9$ , etc. Show that this sequence satisfies a linear recurrence with constant coefficients, so that  $f(-1), f(-2), f(-3), \dots$  have natural values. Interpret these values combinatorially.