1. There is a unique polynomial of degree \( d \) such that \( f(k) = 2^k \) for \( k = 0, 1, \ldots, d \). What is \( f(d + 1) \)? What is \( f(-1) \)?

2. One basis for the space of polynomials of degree less than \( d \) is the monomial basis \( 1, t, t^2, \ldots, t^{d-1} \). Another is the shifted monomial basis \( 1, (t+1), (t+1)^2, \ldots, (t+1)^{d-1} \). Call these bases \( u_1, \ldots, u_d \) and \( v_1, \ldots, v_d \) respectively.

   (a) Derive a formula for the entries of the change-of-basis matrix \( M \) expressing the \( u_i \)'s as linear combinations of the \( v_j \)'s.

   (b) Derive a formula for the entries of the change-of-basis matrix \( N \) expressing the \( v_j \)'s as linear combinations of the \( u_i \)'s.

   (c) From the description of \( M \) and \( N \) as basis-change matrices, we know that \( MN = NM = I \). Forgetting for the moment what \( M \) and \( N \) mean, rewrite the assertions \( MN = NM = I \) as binomial coefficient identities, and prove them either algebraically or bijectively.