Outline of Topics for Final Exam - Discrete I - Spring 2020

Notes:

- "Know basic terminology" means that students should have at least a superficial understanding of the given terminology. For example, this might mean that they can identify when a given object satisfies a definition or doesn't (at least for sufficiently simple examples), and they should be able to justify their answers in a reasonable way. Also, they should know the notation corresponding to the terminology.

Chapter 1: Set Theory

Section 1.1: Set Notation and Relations

- Know basic terminology: set, element, finite set, cardinality, subset, proper subset, set equality
- If a set is given in set-builder notation, write down the elements of the set.
- Express a given set in set-builder notation.
- Know the standard symbols for sets: $\mathbb{P}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$

Section 1.2: Basic Set Operations

- Know basic terminology: disjoint sets
- Given sets, perform any combination of the following set operations: union, intersection, complement of A relative to B, complement of A

Section 1.3: Cartesian Products and Power Sets

- Compute the Cartesian product of sets.
- Given a set A, compute A^2, A^3, etc.
- Compute the power set of a set.

Section 1.4: Binary Representations of Positive Integers

- Given an integer's binary representation, convert to the decimal representation.
- Use Algorithm 1.4.2 to convert a positive integer to binary.

Section 1.5: Summation Notation and Generalizations

- Do computations involving summation notation and generalizations to products and set operations.

Chapter 2: Combinatorics

Section 2.1: Basic Counting Techniques - The Rule of Products

- Apply the rule of products to basic counting problems.

Section 2.2: Permutations

- Know basic terminology: permutations, factorial
- Recognize when permutations occur in counting problems, and use the appropriate formula to compute the number of permutations.

Section 2.3: Partitions of Sets and the Law of Addition

- Know basic terminology: partition

Section 2.4: Combinations and the Binomial Theorem

- Use the binomial coefficient to compute the number of k-element subsets of an n-element set.
- Use the binomial coefficient to compute of bit strings of length n with exactly k ones. Be able to use the same idea on analogous problems (e.g. counting the number of sequences of n coin flips with exactly k heads).
- Use the binomial coefficient and rule of products in counting problems. Examples:
  - Counting the number of strings with exactly k_1 of one letter, exactly k_2 of a second letter, etc. (e.g. How many different strings can be formed by rearranging the letters in the word CALCULUS?)
  - Counting the number of different groups with m_1 elements from one set and m_2 elements from a second set. (e.g. If a class contains four seniors and six juniors, how many groups of five students can be formed which contain three seniors and two juniors?)
- Use the binomial theorem to compute the expansion of $(x+y)^n$ and related expressions (e.g. $(4a-3b)^5$). Be able to extract a specific term from this expansion.

Chapter 3: Logic

Section 3.1: Propositions and Logical Operators

- Know the basic logical operators and their truth tables: negation ($\neg$), conjunction ($\wedge$), disjunction ($\vee$), conditional ($\rightarrow$), biconditional ($\leftrightarrow$)
- Given a statement in English, express the statement symbolically in terms of propositions and logical operators.
- Given a symbolic statement involving propositions and logical operators, translate the statement into English.
- Given a conditional statement, write the contrapositive and converse.

Section 3.2: Truth Tables and Propositions Generated by a Set

- Write down the truth table for a compound proposition.

Section 3.3: Equivalence and Implication

- Know basic terminology: tautology, contradiction, equivalence, implication.
- Know the notation for tautology (1) and contradiction (0).
- Determine if a proposition is a tautology, contradiction, or neither. Justify the answer using a truth table.
- Given two propositions, determine if one proposition implies the other. Justify the answer using a truth table.
- Given two propositions, determine if the propositions are equivalent. Justify the answer using a truth table.

Section 3.4: Laws of Logic

- Understand the laws of logic. Justify why they work using truth tables. (Memorizing the laws is not required.)

Section 3.5: Mathematical Systems and Proofs

- Write direct and indirect proofs in 3-column format.

Section 3.6: Propositions over a Universe

- Know basic terminology: Proposition over a universe and the corresponding concepts of tautology, contradiction, equivalence, and implication.
- Determine the truth set of a proposition over a universe.
- Use truth sets to determine if one proposition implies another proposition, or if two propositions are equivalent.

Section 3.7: Mathematical Induction

- Prove statements of the form "For all positive integers n, $p(n)$ is true" using induction.

Section 3.8: Quantifiers

- Know basic terminology: existential quantifier, universal quantifier
- Write statements symbolically using quantifiers. Translate symbolic statements involving quantifiers into English.
- Write negations of quantified statements using the rules given in Section 3.8.3.

Section 3.9: A Review of Methods of Proof

- Write simple direct and indirect proofs. (For example, problems 1, 2, and 5 from this section.)

Chapter 4: More on Sets

Section 4.1: Methods of Proof for Sets

- Write proofs using the definitions to show that one set is a subset of another set, or to show that two sets are equal.

Section 4.2: Laws of Set Theory

- Be familiar with the laws of set theory.
- Be able to use the laws of set theory to prove new equalities (e.g. problems 3-4).

Section 4.3: Minsets

- Given a finite list of subsets of a universe, list all minsets. Know that the minsets form a partition (Theorem 4.3.8).

Section 4.4: The Duality Principle

- Given a set equality, state the dual (e.g. problem 1).

Chapter 5: Introduction to Matrix Algebra

Section 5.1: Basic Definitions and Operations

- Know basic terminology: Matrix, entry, order, matrix equality, square matrix
- Perform any combination of the following operations on matrices: scalar multiplication, addition, subtraction, multiplication, matrix powers

Section 5.2: Special Types of Matrices

- Know basic terminology: diagonal matrix, identity matrix, matrix inverse
- Be able to verify if a matrix is the inverse of another matrix using the definition.

Section 5.3: Laws of Matrix Algebra / Section 5.4: Matrix Oddities

- Be familiar with the laws of matrix algebra and "matrix oddities" (Observation 5.4.2).
- Be familiar with the similarities and differences between elementary algebra and matrix algebra (e.g. section 5.4.1).
- Be able to produce examples of matrices which satisfy the "matrix oddities."

Chapter 6: Relations

Section 6.1: Basic Definitions

- Know basic terminology: relation from A into B, relation on set A
- Understand the *divides* relation on $\mathbb{Z}$.
- Visualize a relation using a graph (e.g. Figure 6.1.6).
- Compute the composition of relations.

Section 6.2: Graphs of Relations on a Set

- Draw digraphs for relations.
- Given the digraph for a relation, write down the set of ordered pairs for the relation.

Section 6.3: Properties of Relations

- Know the definitions of the following properties: Reflexive, symmetric, antisymmetric, transitive, partial ordering, equivalence relation
- Determine if a given relation has a certain property or not.
- Draw the Hasse diagram for a partially ordered set.

Section 6.4: Matrices of Relations

- Represent a relation using an adjacency matrix.
- Use the adjacency matrix of a relation to find the set of ordered pairs for the relation.
- Do computations using Boolean arithmetic.
- Compute the adjacency matrix of a composition of relations using Theorem 6.4.6. Use this computation to list the ordered pairs in the composition.

Section 6.5: Closure Operations on Relations

- Compute the transitive closure of a relation using Warshall's Algorithm (Algorithm 6.5.8).

Chapter 7: Functions

Section 7.1: Definition and Notation

- Know basic terminology: function, domain, codomain, image, range

Section 7.2: Properties of Functions

- Know the definitions for the following terminology: injective, surjective, bijective.
- Identify when a function is injective, surjective, and/or bijective.

Section 7.3: Function Composition

- Know basic terminology: function equality, identity function
- Compute the composition of functions.
- Compute the inverse of a function.
- If f is a function whose inverse f^{-1} exists, then $f \circ f^{-1}$ and $f^{-1} \circ f$ are equal to the appropriate identity functions.

Chapter 8: Recursion and Recurrence Relations

Section 8.1: The Many Faces of Recursion / Section 8.2: Sequences

- Know basic terminology: Sequences
- Given a recurrence relation with initial conditions, compute additional terms in the sequence.

Section 8.3: Recurrence relations

- Solve recurrence relations using Algorithm 8.3.12.

Section 8.4: Some Common Recurrence Relations

- Solve simple recurrence relations by inspecting the pattern (e.g. Example 8.4.1 or problem 1).