Practice midterm exam for 92.321, Spring 2016 (75 minutes)

Note: In the actual exam, you will be given a workbook that includes space for writing down your answers and showing all your work; you will have two sides of a sheet of paper for each problem.

Problem 1:
Fill in the truth table below for the proposition

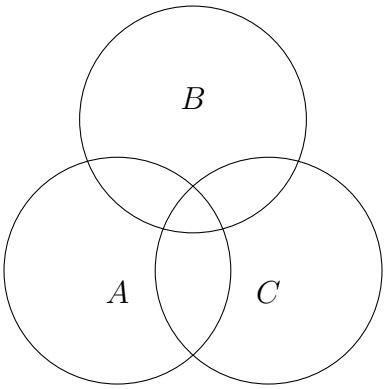
$$((P \to Q) \wedge (Q \to P)) \to (P \ \vee \neg Q)$$

and its constituent sub-propositions.

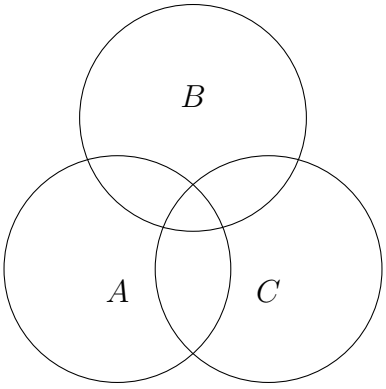
P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \wedge (Q \to P)$	$\neg Q$	$P \vee \neg Q$	$((P \to Q) \wedge (Q \to P)) \to (P \vee \neg Q)$
0	0						
0	1						
1	0						
1	1						

Is the proposition $((P \to Q) \wedge (Q \to P)) \to (P \ \vee \neg Q)$ a tautology? Is it a contradiction? Is it neither? Explain.

Problem 2:
(a) In the Venn diagram below, shade in the region that corresponds to $S = (A \cap B) \cup C$.



(b) In the Venn diagram below, shade in the region that corresponds to $T = (A \cup B) \cap C$.



(c) Is one of the two sets S, T a subset of the other? Explain.

Problem 3:
A chaperone takes five students on a field trip. For snacks, the chaperone has brought two (identical) apples, two (identical) bananas, and a pear. In how many ways can the snacks be distributed so that each student gets exactly one?

Problem 4:
Expand $(2x - y)^4$ using the binomial theorem.

Problem 5:
Use induction to prove that $1 + 3 + 9 + ... + 3^n = (3^{n+1} - 1)/2$ for all integers $n \geq 0$. Make sure to format your proof in the standard way, showing both the base case and the induction step.